SURVEY OF INDIA

DEPARTMENTAL PAPER No. 13

NOTES
ON
SPHERICAL TRIGONOMETRY ASTRONOMY, ETC.,

COMPILED BY
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(From Notes by the late J. Eccles Esq. M.A. & other sources.)

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		h	m	s
L. M. T. = Difference converted into M. T. by table IV of N.A. for 1931 p. 711, or by 23 Sur.	=	19	42	13
Correction for difference between <u>1</u> 5h 12m 13s, and Standard <u>1</u> 5h. 30m.	=		+ 17	47
Standard Time of observation	=	20	00	00

Example 2 (See para 25)

Required to find the Sidereal Time of Local Mean Noon at Dehra Dūn on 1st August 1931. (Para 25 of D.P. 13).

			h	m	s
Sidereal Time from N.A. p. 14 at 0 hrs. on 31st July 1931		= a =	20	30	37.01
Sidereal Time " " 0 hrs. 1st August 1931		= b =	20	34	33.57
Change in 24 hours	= (a-b)	=	- 0	3	56.56
Change in 5h 12m 13s	= 5.2h approx.	=	c.		
	$= \frac{(a-b) \times 5.2}{24}$	=	-51.25		

Then b + c (algebraic sum) = Sidereal Time

		h	m	s
of Local Mean Midnight (0 hrs.)	=	20	33	42.32
Sidereal Equivalent of 12 hrs. from Table III of N.A. for 1931 p. 710	=	12	1	58.28
Sidereal Time of Local Mean Noon	=	8	35	40.60

Example 3. (See para 27).

(a) Required to find the Local Sidereal time of an observation made at Dehra Dūn at 20 hrs. Standard Time on 1st August 1931. (Para 27 of D.P. 13).

		h	m	s
Standard Time	=	20	0	0
For Difference between 1h 5m 12m 13s of Dehra and Standard 5h 30m.	=	-	17	47
L. M. T.	=	19	42	13
Sidereal Equivalent of L.M.T. from Table III, N.A. for 1931 p. 710, or from SP Sur.	=	19	45	27.21
S.T. of L.M. midnight (0 hrs.)	=	20	33	42.32
L.S.T. of observation	=	16	19	9.53

(b) Conversely, given L.S.T. of observation 16h 19m 9.53s, to find Standard Time of observation at Dehra Dūn on 1st August 1931.

		h	m	s
L.S.T. of observation	=	16	19	9.53
Sidereal Time of L.M. midnight (0 hrs.)		20	33	42.32
Difference	=	19	45	27.21

NOTES ON SPHERICAL TRIGONOMETRY

Spherical Trigonometry deals with triangles whose sides are portions of <u>great circles</u>.

Example 1. (See para 24)

Near the Walker Observatory, Dehra Dūn, longitude 78°-3'-15", the Local Apparent Time on 1st August 1931 is 20 hours, what is the Local Mean Time and what is the Standard Time? (Para 24 of D.P. 13).

		h	m	s	
L 78°3' 15"	=	5	12	13	E in time

Addendum to Departmental Paper No.13.

From 1931 the form of the Nautical Almanac has been changed. Except in the Abridged edition, the grouping of data month by month into pages I, II .... XII is no longer followed: instead, the different data are now tabulated separately, and are given continuously for the whole year. The ephemeris of the sun is given in pages 6-54, and that of the moon in pages 55-179.

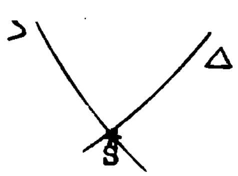
Another important change, as explained in the Preface of the 1931 Almanac, is that all quantities tabulated at regular intervals of 24 hours are now given for midnight (0 hrs.) at Greenwich instead of noon (12 hrs.) as formerly.

A few of the examples of Departmental Paper No.13 are here recomputed with data from N.A. 1931 to illustrate the new arrangement.

L.M.T. required = 20 6 15.48

	h	m	s
Difference between longitude of Dehra and 5h 30m for standard meridian =	5	12	13
	+ 0	17	47
Standard Time required =	20	24	02.48

the altitude of sun or star observed.



PS, the N. Polar Distance Δ,

$\Delta = 90^\circ \pm \delta$, δ being the declination of sun or star observed.

Also the angle at P is the hour angle t, and the angle at Z, the Azimuth (or true bearing) A. These terms are more fully explained

NOTES ON SPHERICAL TRIGONOMETRY

Spherical Trigonometry deals with triangles whose sides are portions of <u>great circles</u>.

Example 1. (See para 24)

Near the Walker Observatory, Dehra Dūn, longitude 78°-3'-15", the Local Apparent Time on 1st August 1931 is 20 hours, what is the Local Mean Time and what is the Standard Time? (Para 24 of D.P. 13).

L 78° 3' 15" = 5h 12m 13s E in time

G.A.T. at 20 hours at Dehra Dūn = 20h − 5h 12m 13s = 14h 47m 47s

The equation of Time for 14h 47m 47s at Greenwich is required.

Equation of Time from p. 14 of N.A. for 1931

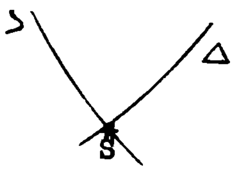
			m	s
at 0 hrs.	=	+	6	15.26
Change in 14.8h approx. at .120s per hour.	=	−	0	1.78
Equation of Time required.	=	+	6	13.48

			h	m	s
L. A. T.	=		20	0	0
Equation of time	=	+		6	13.48
L.M.T. required	=		20	6	13.48
Difference between 5h 12m 13s longitude of Dehra and 5h 30m for standard meridian	=	+	0	17	47
Standard Time required	=		20	24	00.48

the altitude of sun or star observed.

PS, the N. Polar Distance △,

$\triangle = 90^\circ \pm \delta$, δ being the declination of sun or star observed.



Also the angle at P is the hour angle t, and the angle at Z, the Azimuth (or true bearing) A. These terms are more fully explained

NOTES ON SPHERICAL TRIGONOMETRY

Spherical Trigonometry deals with triangles whose sides are portions of great circles.

A great circle on a sphere is a circle traced out by the intersection of the sphere by a plane passing through its centre. If the plane does not pass through the centre of the sphere, its intersection with the sphere is called a small circle.

The necessity for the application of Spherical Trigonometry arises (1) in astronomical work and also (2) in survey work on the earth's surface, when the sides of a triangulation are of an extensive size. In the latter case however, the actual use of Spherical Trigonometry is generally obviated by the use of Legendre's Theorem(vide p.13 of these notes),which enables us to treat the solution of a spherical triangle as that of a plane one, after applying Spherical Excess.

(1) Application to astronomical work. The object - sun or star - to which an astronomical observation is taken, (S in the figure), forms with the zenith (Z),and elevated pole (P),a spherical triangle SPZ, whose sides are arcs of great circles of the celestial sphere. These sides are PZ, the colatitude γ,

$\gamma = (90^\circ - \lambda)$, λ being the latitude.

ZS, the zenith distance ζ,

$\zeta = (90^\circ - h)$, h being the altitude of sun or star observed.

PS, the N.Polar Distance Δ,

$\Delta = 90^\circ \pm \delta$, δ being the declination of sun or star observed.

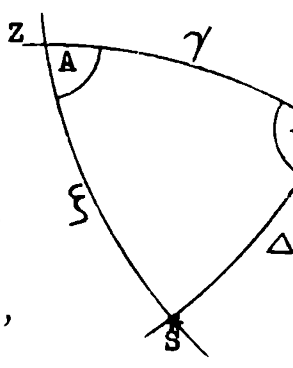


Fig. 1

Also the angle at P is the hour angle t, and the angle at Z, the Azimuth(or true bearing) A. These terms are more fully explained in the Notes on Astronomy, but for the present it may be merely stated that, if any three of these elements be known, the other three can be deduced. Thus we can determine time(or error of a watch or chronometer), latitude or azimuth, certain elements being known or taken from the Nautical Almanac etc. and the others actually observed.

(2) Application to Survey work on the earth's surface. The curvature of the earth's surface is not an appreciable factor in a small survey, but when its limits are extended, curvature has to be taken into account. It is therefore necessary to use Spherical Trigonometry instead of the more familiar Plane Trigonometry, though Legendre's theorem in most cases enables us to treat the solution of triangles by Plane Trigonometry(as already stated, vide also p 13.) A "straight" line in Spherical Trigonometry(e.g.-the side of a triangle measured on the earth's surface) is represented by the arc of a great circle. Hence a parallel of latitude, which is a small circle, does not correspond with a "straight" measurement, nor does a movement due East or West correspond with a movement along a "straight" line on the earth's surface. A triangle set out on the earth's surface with "straight" sides is termed a "spherical triangle", its sides being arcs of great circles.

In Figure 2, C is the centre of the sphere and P the pole, DEF a section made by a plane at a constant distance CN from the centre and perpr to CP. Then, as CN = constant & CD = constant = radius

$DN^2 = CD^2 - CN^2$ = constant
DN = constant
Therefore D is on a circle
Similarly PD is constant
Hence the arc PD is constant

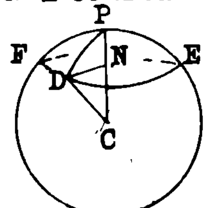


Fig 2

It may be here noted that only one great circle can be drawn through 2 points on a sphere, unless the 2 points are the ends of a diameter, when the number is infinite.

In the figure C and P are the centre and pole of the sphere, as before, AB a portion of the equator, intercepted between two meridians AP,BP, ab being their corresponding intercept on a small circle of latitude λ i.e:- the angle bCB is λ .

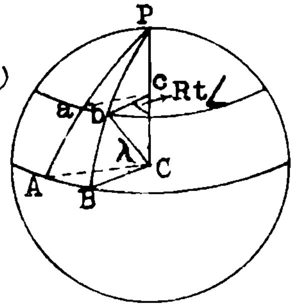


Fig.3

Now $\angle$ abc = $\angle$ABC   bC = kC (being the radii)

$$\frac{AB}{ab} = \frac{BC}{bc} = \frac{bC}{bc} = \frac{1}{\sin bCc}$$ ∠bcC is rt ∠

ab = ABsin bCc
  = ABsin(90° − bCB)
  = ABcosλ, (λ the latitude) .....(1)

As already stated, the sides of spherical triangles are the arcs of great circles. The angles between two such curved sides is the angle between the tangents, as explained in Fig.4 below. The angle C of the triangle ABC is the angle TCt, where TC is perpr to OC, and tC also perpr to OC, O being the centre of the sphere. i.e:- TCt is the angle between the planes OAC, OBC.
In a spherical triangle, it may also be noted that the arc AB is proportional to the angle AOB, and therefore instead of speaking of AB as a length, it is legitimate to represent it by the angle AOB.

Thus in Spherical Trigonometry the $\angle$AOB represents the side c, $\angle$AOC the side b, and $\angle$COB the side a , these being the angles subtended by these sides at the centre of the sphere.
We thus speak of sin a or cos b or tan c etc., meaning sin COB or cos AOC or tan AOB etc,. Also to convert such angles, if in circular measure, to seconds, it is necessary to divide them by the circular measure of 1", or approx by the sine of 1", or multiply them by cosec1".

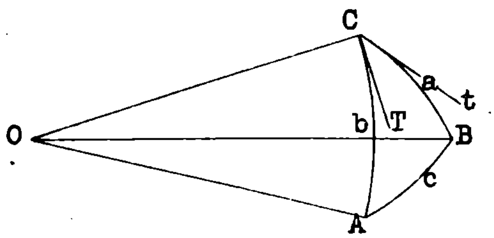


Fig.4.

RIGHT-ANGLED SPHERICAL TRIANGLES

Let ABC be a right-angled spherical triangle with the right angle at C. Take any point P in OA

Draw PM perpr to OC

MN perpr to OB

Join PN

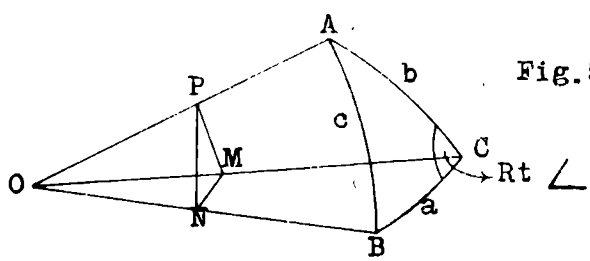


Fig.5.

Since C is a right angle, the plane OAC is perpr to the plane OBC, and MP is drawn from a point in the line of intersection perpr to the line of intersection OC of the two planes,

Then PM is perpr to the plane OBC and therefore perpr to MN

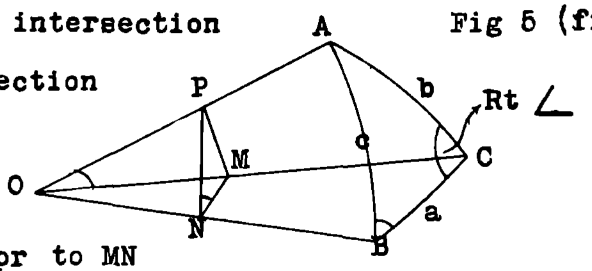


Fig 5 (from p.3)

Then

$$PN^2 = PM^2 + MN^2$$
$$= OP^2 - OM^2 + MN^2$$
$$= OP^2 - ON^2$$

Therefore ∠ PNO is a right angle and PN is perpr to OB

As then PN and MN are both perpr to OB, the angle PNM = angle B.

Now $\frac{PM}{PN} = \frac{PM}{OP} \times \frac{OP}{PN}$ $\frac{\sin POM}{\sin PON}$ or $\frac{\sin AOC}{\sin AOB}$

$\sin B = \frac{\sin b}{\sin c}$, as we speak of the angles AOC,AOB, subtended at the centre O as represented by the sides b,c, respectively(vide p page: ,note side-lined)

Therefore $\sin b = \sin c \sin B$ ⎫
Similarly $\sin a = \sin c \sin A$ ⎭ ....(2)a

and thus we have, as C is 90° and $\sin C = 1$

$$\frac{\sin A}{\sin a} = \frac{\sin B}{\sin b} = \frac{\sin C}{\sin c} \quad \text{........(2)}$$

Again $\frac{PM}{MN} = \frac{PM}{OM} \times \frac{OM}{MN}$

$\tan B = \frac{\tan b}{\sin a}$

$\tan b = \sin a \tan B$ ⎫
Similarly
$\tan a = \sin b \tan A$ ⎭ ..(3)

$\frac{ON}{OP} = \frac{ON}{OM} \times \frac{OM}{OP}$

$\cos c = \cos a \cos b$ ...(4)

$\frac{MN}{PN} = \frac{MN}{ON} \times \frac{ON}{PN}$

$\cos B = \tan a \cot c$ ⎫
Similarly
$\cos A = \tan b \cot c$ ⎭ ..(5)

$\cos B = \sin b \tan A \cot c$

$= \frac{\sin b}{\sin c} \tan A \cos c$

$= \sin B \tan A \cos c$ ..(6)

∴ $\cot B = \tan A \cos c$ ⎫
Similarly
$\cot A = \tan B \cos c$ ⎭ .....(7)

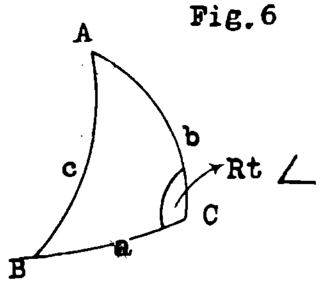
$\cos A = \frac{\sin b}{\cos b} \times \frac{\cos c}{\sin c}$

$= \frac{\sin B \cos a \cos b}{\cos b}$

$= \sin B \cos a$ ... (8)

The formulae of the preceding page may be easily remembered by <u>Napier's Rules</u>;which are as follows:-

Fig.6



Leave out C, the right angle, in the triangle ABC, shown in the diagram, and consider the parts a,b,c,A,B. Whenever you come to an angle or the hypotenuse, write $(90^\circ - \text{angle})$ or $(90^\circ - \text{hypotenuse})$, and we have:-

<u>Sin(any part) = Product of cosines of opposite parts</u>;

<u>Sin(any part) = Product of tangents of adjacent parts</u>

Thus $\sin a = \cos(90^\circ - A)\cos(90^\circ - c)$

$= \sin A \sin c$, which corresponds with results (2)a,(2), on previous page

$\sin a = \tan(90^\circ - B)\tan b$

$= \cot B \tan b$,which corresponds with result (3), p.page

$\sin b = \cos(90^\circ - B)\cos(90^\circ - c)$

$= \sin B \sin c$,which corresponds with (2)a,(2), p.page

$\sin b = \tan(90^\circ - A)\tan a$

$= \cot A \tan a$, which corresponds with (3), p.page

$\sin(90^\circ - A) = \cos(90^\circ - B)\cos a$

$\cos A = \sin B \cos a$, which corresponds with (8), p.page

$\sin(90^\circ - A) = \cos A$

$= \tan b \tan(90^\circ - c)$

$= \tan b \cot c$, which corresponds with (5), p.page

$\sin(90^\circ - c) = \cos a \cos b$

$\cos c = \cos a \cos b$, which corresponds with (4), p.page

$\sin(90^\circ - c) = \tan(90^\circ - A)\tan(90^\circ - B)$

$\cos c = \cot A \cot B$, which corresponds with (7), p.page

$\sin(90^\circ - B) = \cos b \cos(90^\circ - A)$

$\cos B = \cos b \sin A$, which corresponds with (8), p.page, the formula being for cos B instead of cos A

$\sin(90^\circ - B) = \tan a \tan(90^\circ - c)$

$\cos B = \tan a \cot c$, which corresponds with (5), p.page.

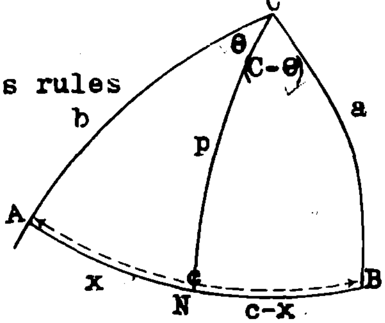
ORDINARY TRIANGLES

In the figure draw CN perpr to AB and call it p.

Call AN x , then BN $= c-x$

Call angle ACN θ, then BCN $= C-\theta$

Fig 7



$$\begin{aligned}\sin p &= \cos(90^\circ-a)\cos(90^\circ-B) \text{ by Napier's rules}\\ &= \sin a \sin B\\ \sin p &= \cos(90^\circ-b)\cos(90^\circ-A)\\ &= \sin b \sin A\end{aligned}$$

Therefore

$$\sin a \sin B = \sin b \sin A$$

Similarly

$$\sin c \sin A = \sin a \sin C$$

Hence

$$\frac{\sin A}{\sin a} = \frac{\sin B}{\sin b} = \frac{\sin C}{\sin c} \quad \text{......(9)}$$

$$\begin{aligned}\sin(90^\circ-a) &= \cos p \cos(c-x) \text{ by Napier's rules}\\ \cos a &= \cos p\,(\cos c \cos x + \sin c \sin x)\\ &= \cos c \cos p \cos x + \sin c \sin x \cos p\end{aligned}$$

But

$$\begin{aligned}\cos b &= \sin(90^\circ-b)\\ &= \cos p \cos x\\ \sin x &= \tan p \tan(90^\circ-A)\\ &= \tan p \cot A\\ &= \frac{\sin p \cos A}{\sin A \cos p}\end{aligned}$$

Therefore

$$\begin{aligned}\sin x \cos p &= \cos A \frac{\sin p}{\sin A}\\ &= \cos A \sin b\end{aligned}$$

Hence

$$\begin{aligned}\cos a &= \cos b \cos c + \sin b \sin c \cos A\\ \cos b &= \cos a \cos c + \sin a \sin c \cos B \quad \text{.........(10)}\\ \cos c &= \cos a \cos b + \sin a \sin b \cos C\end{aligned}$$

$$\begin{aligned}\sin(90^\circ-B) &= \cos(90^\circ-(C-\theta))\cos p \text{ by Napier's rules}\\ \cos B &= \sin(C-\theta)\cos p\\ &= \sin C \cos\theta \cos p - \cos C \sin\theta \cos p\end{aligned}$$

But

$$\begin{aligned}\sin(90^\circ-A) &= \cos(90^\circ-\theta)\cos p\\ &= \sin\theta \cos p\\ \sin(90^\circ-\theta) &= \tan p \tan(90^\circ-b)\\ \cos\theta &= \tan p \cot b\\ \text{i.e.}\quad \cos\theta\cos p &= \frac{\sin p}{\sin b}\cos b\\ &= \sin A \cos b\end{aligned}$$

Hence

$$\begin{aligned}\cos B &= -\cos A \cos C + \sin A \sin C \cos b\\ \cos C &= -\cos A \cos B + \sin A \sin B \cos c \quad \text{.........(11)}\\ \cos A &= -\cos B \cos C + \sin B \sin C \cos a\end{aligned}$$

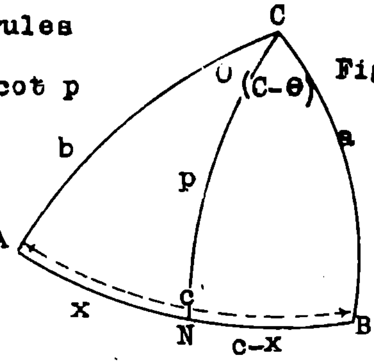


Fig.7

$\sin(c-x) = \tan p \tan(90^\circ - B)$ by Napier's rules

$\sin c \cos x - \cos c \sin x = \tan p \cot B$

$\cot B = \sin c \cos x \cot p - \cos c \sin x \cot p$

But $\sin x = \tan p \cot A$

Therefore

$$\cot B = \sin c \cos x \cot p - \cos c \cot A$$

$$= \frac{\sin c \cos b}{\sin b \sin A} - \frac{\cos c \cos A}{\sin A}$$

or

$$\cot B \sin A = \sin c \cot b - \cos c \cos A$$

and so we have:-

$$\left.\begin{array}{l} \cos c \cos A = \sin c \cot b - \sin A \cot B \\ \cos c \cos B = \sin c \cot a - \sin B \cot A \end{array}\right\} \ldots\ldots\ldots\ldots(12)$$

$$\left.\begin{array}{l} \cos a \cos C = \sin a \cot b - \sin C \cot B \\ \cos a \cos B = \sin a \cot c - \sin B \cot C \end{array}\right\} \ldots\ldots\ldots\ldots(13)$$

ERRATA TO DEPARTMENTAL PAPER NO. 15.

Spherical Trigonometry.

Page 7 line 20 from top, read the equation as

$\cos a = \cos b \cos c + \sin b \sin c \cos A$

cos(middle ...

sin(middle side) cot(other side) — sin(middle angle)x cot(other angle) ....(15)

$$\cos a = \cos b \cos c + \sin b \sin c \cos A \ \ldots\ldots \text{ from } (10)$$ (1)

$$\cos A = \frac{\cos a - \cos b \cos c}{\sin b \sin c}$$

$$1 - \cos A = \frac{\cos b \cos c + \sin b \sin c - \cos a}{\sin b \sin c}$$

$$= \frac{\cos(b-c) - \cos a}{\sin b \times \sin c}$$

$$2 \sin^2 \frac{A}{2} = \frac{2 \sin\left(\frac{a+b-c}{2}\right) \sin\left(\frac{a-b+c}{2}\right)}{\sin b \sin c}$$

Similarly

$$1 + \cos A = \frac{\cos a - \cos(b+c)}{\sin b \sin c}$$

$$2 \cos^2 \frac{A}{2} = \frac{2 \sin\left(\frac{a+b+c}{2}\right) \sin\left(\frac{b+c-a}{2}\right)}{\sin b \sin c}$$

Let $a+b+c = 2s$, then $\frac{a+b-c}{2}$ & $\frac{a-b+c}{2} = (s-c)$ and $(s-b)$ respectively, also $\frac{b+c-a}{2} = s-a$

therefore

$$\sin\frac{A}{2} = \sqrt{\frac{\sin(s-b) \sin(s-c)}{\sin b \sin c}} \qquad \cos\frac{A}{2} = \sqrt{\frac{\sin s \sin(s-a)}{\sin b \sin c}} \quad \ldots(16)$$

$$\sin\frac{B}{2} = \sqrt{\frac{\sin(s-a) \sin(s-c)}{\sin a \sin c}} \qquad \cos\frac{B}{2} = \sqrt{\frac{\sin s \sin(s-b)}{\sin a \sin c}} \quad \ldots(17)$$

$$\sin\frac{C}{2} = \sqrt{\frac{\sin(s-a) \sin(s-b)}{\sin a \sin b}} \qquad \cos\frac{C}{2} = \sqrt{\frac{\sin s \sin(s-c)}{\sin b \sin a}} \quad \ldots(18)$$

$\sin(c-x) = \tan p \tan(90^{\circ} - B)$ by Napier's rules
$\sin c \cos x - \cos c \sin x = \tan p \cot B$
$\cot B = \sin c \cos x \cot p - \cos c \sin x \cot p$
But $\sin x = \tan p \cot A$
Therefore
$\cot B = \sin c \cos x \cot p - \cos c \cot A$

$$= \frac{\sin c \cos b}{\sin b \sin A} - \frac{\cos c \cos A}{\sin A}$$

or

$\cot B \sin A = \sin c \cot b - \cos c \cos A$

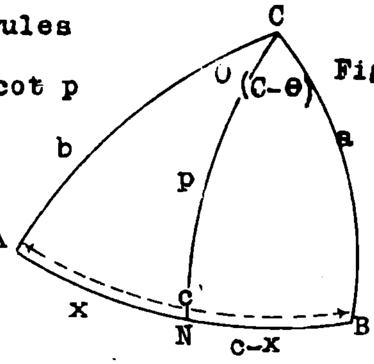


Fig.7

and so we have:-

$$\left.\begin{aligned}\cos c \cos A &= \sin c \cot b - \sin A \cot B\\ \cos c \cos B &= \sin c \cot a - \sin B \cot A\end{aligned}\right\} \quad \ldots\ldots\ldots\ldots(12)$$

$$\left.\begin{aligned}\cos a \cos C &= \sin a \cot b - \sin C \cot B\\ \cos a \cos B &= \sin a \cot c - \sin B \cot C\end{aligned}\right\} \quad \ldots\ldots\ldots\ldots(13)$$

$$\left.\begin{aligned}\cos b \cos A &= \sin b \cot c - \sin A \cot C\\ \cos b \cos C &= \sin b \cot a - \sin C \cot A\end{aligned}\right\} \quad \ldots\ldots\ldots\ldots(14)$$

Formulae (12) (13) (14) may be summarized thus:-

Take any four consecutive parts, 2 sides and 2 angles.

Then

cos(middle side)xcos(middle angle) =

sin(middle side) cot(other side) − sin(middle angle)x
cot(other angle) ....(15)

$\cos a = \cos b \cos c + \sin b \sin c \cos A$ .... from (10) ①

$$\cos A = \frac{\cos a - \cos b \cos c}{\sin b \sin c}$$

$$1 - \cos A = \frac{\cos b \cos c + \sin b \sin c - \cos a}{\sin b \sin c}$$

$$= \frac{\cos(b-c) - \cos a}{\sin b \times \sin c}$$

$$2 \sin^2 \frac{A}{2} = \frac{2 \sin\left(\frac{a+b-c}{2}\right)\sin\left(\frac{a-b+c}{2}\right)}{\sin b \sin c}$$

Similarly

$$1 + \cos A = \frac{\cos a - \cos(b+c)}{\sin b \sin c}$$

$$2 \cos^2 \frac{A}{2} = \frac{2\sin\left(\frac{a+b+c}{2}\right)\sin\left(\frac{b+c-a}{2}\right)}{\sin b \sin c}$$

Let $a+b+c = 2s$, then $\frac{a+b-c}{2}$ & $\frac{a-b+c}{2}$ = (s−c) and (s−b) respectively, also $\frac{b+c-a}{2} = s-a$

therefore

$$\sin\frac{A}{2} = \sqrt{\frac{\sin(s-b) \sin(s-c)}{\sin b \sin c}} \qquad \cos\frac{A}{2} = \sqrt{\frac{\sin s \sin(s-a)}{\sin b \sin c}} \quad \ldots(16)$$

$$\sin\frac{B}{2} = \sqrt{\frac{\sin(s-a) \sin(s-c)}{\sin a \sin c}} \qquad \cos\frac{B}{2} = \sqrt{\frac{\sin s \sin(s-b)}{\sin a \sin c}} \quad \ldots(17)$$

$$\sin\frac{C}{2} = \sqrt{\frac{\sin(s-a) \sin(s-b)}{\sin a \sin b}} \qquad \cos\frac{C}{2} = \sqrt{\frac{\sin s \sin(s-c)}{\sin b \sin a}} \quad \ldots(18)$$

By division from the formulae (16),(17),(18), the value of

$$\tan\frac{A}{2} = \sqrt{\frac{\sin(s-b)\sin(s-c)}{\sin s\ \sin(s-a)}}$$
$$\tan\frac{B}{2} = \sqrt{\frac{\sin(s-c)\sin(s-a)}{\sin s\ \sin(s-b)}} \quad \ldots\ldots\ldots (19)$$
$$\tan\frac{C}{2} = \sqrt{\frac{\sin(s-a)\sin(s-b)}{\sin s\ \sin(s-c)}}$$

Again from formula (11) we have

$$\cos a = \frac{\cos A + \cos B\cos C}{\sin B\sin C}$$

$$1-\cos a = \frac{\sin B\sin C - \cos B\cos C - \cos A}{\sin B\sin C}$$

$$= \frac{-\cos(B+C)-\cos A}{\sin B\sin C}$$

$$2\sin^2\frac{a}{2} = \frac{-2\cos\left(\frac{A+B+C}{2}\right)\cos\left(\frac{B+C-A}{2}\right)}{\sin B\sin C}$$

$$\sin^2\frac{a}{2} = \frac{-\cos S\cos(S-A)}{\sin B\sin C} \quad \ldots\ldots(20)$$

where $S = \dfrac{A+B+C}{2}$

Similarly by taking 1+cos a we get

$$\cos^2\frac{a}{2} = \frac{\cos(S-B)\cos(S-C)}{\sin B\sin C} \quad \ldots\ldots(21)$$

From (20) (21) by division we get

$$\tan^2\frac{a}{2} = \frac{-\cos S\cos(S-A)}{\cos(S-B)\cos(S-C)} \quad \ldots\ldots\ldots(22)$$

$$\tan\frac{(A+B)}{2} = \frac{\tan\frac{A}{2} + \tan\frac{B}{2}}{1-\tan\frac{A}{2}\tan\frac{B}{2}}$$

By substituting from (19), after some reduction, we get:-

$$\tan\frac{(A+B)}{2} = \sqrt{\frac{\sin s\ \sin(s-c)}{\sin(s-a)\ \sin(s-b)}} \times \frac{\sin(s-b)+\sin(s-a)}{\sin s - \sin(s-c)}$$

That is

$$\tan\frac{A+B}{2} = \cot\frac{C}{2}\,\frac{2\sin\left(\frac{2s-a-b}{2}\right)\cos\left(\frac{a-b}{2}\right)}{2\cos\left(\frac{2s-c}{2}\right)\sin\frac{c}{2}}$$

$$= \frac{\cos\frac{a-b}{2}}{\cos\frac{a+b}{2}}\cot\frac{C}{2} \quad \ldots\ldots\ldots\ldots(23)a$$

Similarly

$$\tan\frac{A-B}{2} = \frac{\sin\frac{a-b}{2}}{\sin\frac{a+b}{2}}\cot\frac{C}{2} \quad \ldots\ldots\ldots\ldots(23)b$$

Also

$$\tan\frac{a+b}{2} = \frac{\cos\left(\frac{A-B}{2}\right)}{\cos\left(\frac{A+B}{2}\right)}\tan\frac{c}{2}\ldots\ldots\ldots\ldots(24)a$$

$$\tan\frac{a-b}{2} = \frac{\sin\left(\frac{A-B}{2}\right)}{\sin\left(\frac{A+B}{2}\right)}\tan\frac{c}{2} \quad \ldots\ldots\ldots\ldots(24)b$$

Most of the formulae already explained are summarized in Auxiliary Tables (1928) Survey of India Part III p 67. There are however a few variations in some of the formulae for computing purposes, which remain to be explained.
Thus from formula (10) we have as it stands:-

$$\cos a = \cos b \cos c + \sin b \sin c \cos A$$

Transposing $\cos A = \dfrac{\cos a - \cos b \cos c}{\sin b \sin c}$

$$= \frac{\cos(a+\theta)}{\sin b \sin c \cos\theta}, \left[\text{where } \tan\theta = \frac{\cos b \cos c}{\sin a}\right]$$

as on p 67 Aux Tab Part III 1928 Sph △s 2(i) .... (25)

Again $\cos c = \cos a \cos b + \sin a \sin b \cos C$ from (10)

$$= \frac{\cos a \cos(b-\theta)}{\cos\theta}, \text{ where } \tan\theta = \tan a \cos C \ \ldots.(26)$$

If c be not near 90°, take $\cos n = \cos a \cos(b-\theta)$, then

$$\tan^2\tfrac{1}{2}c = \tan\tfrac{1}{2}(n+\theta)\tan\tfrac{1}{2}(n-\theta) \ \ldots.. (27)$$

as on p 67 Aux Tab Part III 1928 Sph △s 3(i)

From formula (15) we have

$$\sin C \cot A = \sin b \cot a - \cos b \cos C$$

$$\text{Hence } \tan A = \frac{\sin a \sin C}{\sin b \cos a - \sin a \cos b \cos C}$$

$$= \frac{\sin \theta \tan C}{\sin(b-\theta)}, \text{ where } \tan \theta = \tan a \cos C \quad \ldots\ldots(28)$$

as on p 67 Aux Tab Part III 1928, Sph $\triangle$s, 3(ii)

In Spherical as in Plane Trigonometry, an Ambiguous Case arises in solving a triangle, in which two sides and the angle opposite one of them are given, e.g.- a, b, A.

We have by formula (10)

$$\cos b \cos c + \sin b \sin c \cos A = \cos a \quad \ldots.(29)$$

To solve this, put $k \sin \theta = \sin b \cos A$

$$k \cos \theta = \cos b$$

Equation (29) becomes

$$k \cos(c-\theta) = \cos a$$

Put $(c-\theta) = \theta'$

$$k \cos \theta' = \cos a$$

$$c = \theta \pm \theta' \quad \ldots\ldots.(30)$$

The auxiliary angle θ is fully determined by (30), being taken between 0° and 180° and always positive, but, as the cosine of an $\angle$ = cosine of the negative of that $\angle$, we can take θ' in (30) as positive or negative, so that $c = \theta \pm \theta'$. There are thus 2 values of c, both of which are admissible, except when $\theta + \theta'$ exceeds 180°, in which case the only solution is $c = \theta - \theta'$, and except when θ' exceeds θ, which would make c negative. In the latter case the only solution is $c = \theta + \theta'$.

Eliminating k from (30), we have for finding c,

$$\tan \theta = \tan b \cos A$$

$$\cos \theta' = \frac{\cos \theta \cos a}{\cos b} \quad \ldots.(31)$$ c.f. Aux Tab Part III 1928 p 67, Sph $\triangle$s, 4(i)

$$c = \theta \pm \theta'$$

When θ' is small, take ~~cos~~ sec p = sec b cos θ

$$\text{Then } \tan^2 \tfrac{1}{2}\theta' = \tan \tfrac{1}{2}(a+p) \tan \tfrac{1}{2}(a-p) \quad \ldots.(32)$$

The other sides and angles are found from formula (9).

From the formula (11)

" 11 " 2 from top, read the numerator as P. 11 of Spherical Trig.
cos **A** + cos **B** cos C

$$= \frac{\sin(A+\theta)}{\sin B \sin C \sin \theta} \quad ..(33)$$

$$\text{where } \cot \theta = \frac{\cos B \cos C}{\sin A}$$

as on p 68, Aux Tab Part III (1928) Sph $\triangle$s 6 (i)

From the formula (11)

$$\cos C = -\cos A \cos B + \sin A \sin B \cos c$$

$$= \frac{\cos A \sin(B-\theta)}{\sin \theta} \quad ......(34)$$

$$\text{where } \cot \theta = \cos c \tan A$$

as on p 68, Aux Tab Part III (1928) Sph $\triangle$s 7 (i)

$$\sin b = \frac{\sin a \sin B}{\sin A}$$ vide p 68, Aux Tab Part III (1928) Sph $\triangle$s 8 (i)

When b is near 90°, take $\sin p = \sin a \sin B$

Then

$$\tan^2(45-\tfrac{1}{2}b) = \frac{\sin^2(45^\circ-\tfrac{1}{2}b)}{\cos^2(45^\circ-\tfrac{1}{2}b)}$$

$$= \frac{1-\cos(90^\circ-b)}{1+\cos(90^\circ-b)}$$

$$= \frac{1-\sin b}{1+\sin b}$$

$$= \frac{1-\dfrac{\sin a \sin B}{\sin A}}{1+\dfrac{\sin a \sin B}{\sin A}}$$

$$= \frac{\sin A - \sin p}{\sin A + \sin p}$$

$$= \frac{2 \cos \frac{(A+p)}{2} \sin \frac{(A-p)}{2}}{[illegible]\ (A+p)\ [illegible]\ (A-p)}$$

" 10 of Ast / Spherical Trig " 3 from bottom, for cos p read sec p.

= cot ½ (A+p) tan ½ (A-p) vide p 68 Aux Tab Part III.1928 Sph $\triangle$s 8 (1).........(35)

From the formula (11)

$$\cos A = -\cos B \cos C + \sin B \sin C \cos a$$

From the formula (11)

$$\cos a = \frac{\cos A + \cos B \cos C}{\sin B \sin C}$$

$$= \frac{\sin(A+\theta)}{\sin B \sin C \sin\theta} \quad ..(33)$$

where $\cot\theta = \dfrac{\cos B \cos C}{\sin A}$

as on p 68, Aux Tab Part III (1928) Sph $\triangle$s 6 (i)

From the formula (11)

$$\cos C = -\cos A \cos B + \sin A \sin B \cos c$$

$$= \frac{\cos A \sin(B-\theta)}{\sin\theta} \quad ......(34)$$

where $\cot\theta = \cos c \tan A$

as on p 68, Aux Tab Part III (1928) Sph $\triangle$s 7 (i)

$\sin b = \dfrac{\sin a \sin B}{\sin A}$ vide p 68, Aux Tab Part III (1928) Sph $\triangle$s 8 (i)

When b is near 90°, take $\sin p = \sin a \sin B$

Then

$$\tan^2(45-\tfrac{1}{2}b) = \frac{\sin^2(45^\circ-\tfrac{1}{2}b)}{\cos^2(45^\circ-\tfrac{1}{2}b)}$$

$$= \frac{1-\cos(90^\circ-b)}{1+\cos(90^\circ-b)}$$

$$= \frac{1-\sin b}{1+\sin b}$$

$$= \frac{1-\dfrac{\sin a \sin B}{\sin A}}{1+\dfrac{\sin a \sin B}{\sin A}}$$

$$= \frac{\sin A - \sin p}{\sin A + \sin p}$$

$$= \frac{2\cos\frac{(A+p)}{2}\sin\frac{(A-p)}{2}}{2\sin(\frac{A+p}{2})\cos(\frac{A-p}{2})}$$

$= \cot\tfrac{1}{2}(A+p)\tan\tfrac{1}{2}(A-p)$ vide p 68 Aux Tab Part III, 1928 Sph $\triangle$s 8 (1).........(35)

From the formula (11)

$$\cos A = -\cos B \cos C + \sin B \sin C \cos a$$

If we put

$$h \sin \theta = \cos B$$

$$h \cos \theta = \sin B \cos a$$

also $C - \theta = \theta'$

We have $h \sin \theta' = \cos A$

$$C = \theta + \theta'$$

Eliminating h, we have

$$\left.\begin{aligned} \cot \theta &= \tan B \cos a \\ \sin \theta' &= \frac{\sin \theta \cos A}{\cos B} \end{aligned}\right\} \quad \ldots\ldots(36)$$

$C = \theta + \theta'$, as on p 51 Aux Tab Part III Sph $\triangle$s 8(ii)

As θ' is determined by its sine, it will have two supplemental values, which will both be added to or both subtracted from θ, according to the sign of $\sin \theta'$, thus giving 2 values of C, except when one of them exceeds 180°, or when one of them is negative.

When θ' is near 90°, take $\cos p = \cos B \operatorname{cosec} \theta$

$$\text{Then } \tan^2(45^\circ - \tfrac{1}{2}\theta') = \tan \tfrac{1}{2}(A+p) \tan \tfrac{1}{2}(A-p)$$

This relation can be easily obtained in a manner similar to the formula (35) on preceding page

The three angles being now known, the other parts can be found by formulae (20) (21) or (22) cf the above para of Aux Tab.

The formula given in p.68 Aux Tab 1928 Part III Sph $\triangle$s 7(iii) is merely a slight variation of formula (15)

By formula (15) we have

$$\cos c \cos B = \sin c \cot a - \sin B \cot A$$

$$\text{or} \quad \cot a \sin c = \cos B \cos c + \cot A \sin B \quad \ldots\ldots(37)$$

$$\text{or} \quad \cot a = \cos B \cot c + \cot A \sin B \operatorname{cosec} c$$

$$= \cot c \cos (B - \theta) \sec \theta \quad \ldots\ldots(38)$$

$$\text{where } \cot \theta = \cos c \tan A$$

The other rule relating any four adjacent parts of the three sides and three angles of a spherical △, given in p. 68 of Aux Tab 1928 Part III Sph △s 7(iii), is merely another way of stating formula (15).

The adjacent parts are here designated A_0 , s_1 , A_1 , s_0.

and if s_0 outer side , A_0 outer ∠

s_1 inner side , A_1 inner ∠

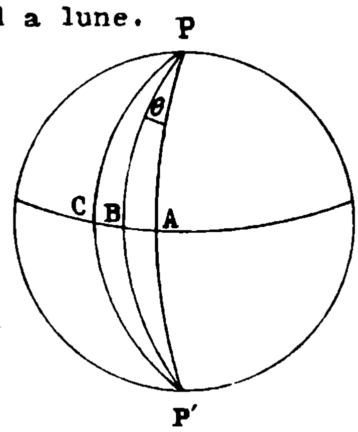
then $\cot s_0 \sin s_1 - \cot A_0 \sin A_1 = \cos s_1 \cos A_1$ .... (39)

SPHERICAL EXCESS

The sum of the angles of a spherical △ exceeds 180° or the sum of the angles of a plane △ by an amount called the spherical excess.

The most important theorem concerning the spherical excess of a triangle is Legendre's Theorem which states that if we have a spherical △ ABC and we make a plane △ A'B'C' sides a',b',c', so that A' = A - $\frac{1}{3}$rd Sph excess, B' = B - $\frac{1}{3}$rd Sph excess, C' = C - $\frac{1}{3}$rd Sph excess, then a = a', b = b', c = c'. This theorem is proved on pages 14 and 15. Two preliminary propositions have however first to be established.

Fig 8



In the figure, the portion PAP'BP is called a lune.

Lunes are to one another as their angles

$$\frac{\text{Lune } \theta}{\text{Lune 4 rt } \angle\text{s}} = \frac{\theta}{\text{4 rt } \angle\text{s}}$$

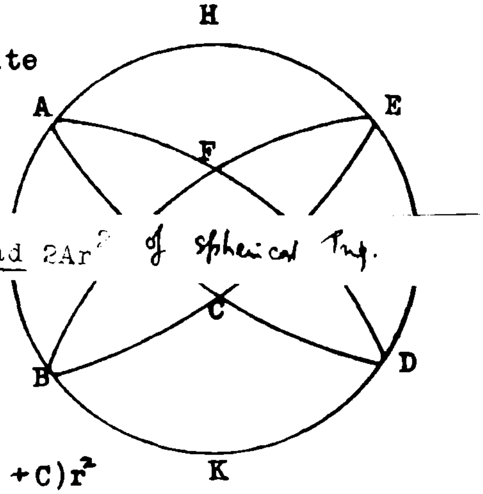
$$\frac{\text{Lune } \theta}{4\pi r^2} = \frac{\theta}{2\pi}$$

$$\text{Lune } \theta = 2 r^2 \theta \quad \text{.............. (40)}$$

In the figure ACDF and BCEF are great circles

It can be seen by symmetry that the triangle ABC = triangle DEF, at opposite sides of the diameter.

Fig 9



Triangle ABC + BCDK = lune A

" 14 " 6 from top, for $2Ar^1$ read $2Ar^2$ of Spherical Trig.

Triangle ABC + [illegible]

$= 2Br^2$

Triangles ABC + CDE $=$ lune C

$= 2Cr^2$

2 (triangle ABC) + hemisphere $= 2(A+B+C)r^2$

2(triangle ABC) $= 2(A+B+C-\pi)r^2$

Triangle ABC $= (A+B+C-\pi)r^2$

$A+B+C-\pi$ is called the Spherical Excess ......(41)

$$\frac{\text{Area of Triangle ABC}}{\text{Area of hemisphere}} = \frac{A+B+C-\pi}{2\pi} = \frac{\text{Sph Excess}}{4 \text{ rt } \angle s} \quad ....(42)$$

Very few sides of a triangulation exceed 100 miles. The circular measure of this is, (taking the earth's radius roughly as 4000 miles)

$\frac{100}{4000} = \frac{1}{40} = \alpha \;\Big|\; \alpha^2 = \frac{1}{1,600} \;\Big|\; \alpha^4 = \frac{1}{2,560,000}$ and it is unnecessary to retain terms smaller than this for our purpose.

Let α, β, γ be the lengths of the sides a, b, c, of the triangle ABC

$$\cos A = \frac{\cos a - \cos b \cos c}{\sin b \sin c}$$

Expanding the right hand side by the expansions for sine and cosine in circular measure we get:-

$$\cos A = \frac{\left(1-\frac{\alpha^2}{2r^2}+\frac{\alpha^4}{24r^4}\right)-\left(1-\frac{\beta^2}{2r^2}+\frac{\beta^4}{24r^4}\right)\left(1-\frac{\gamma^2}{2r^2}+\frac{\gamma^4}{24r^4}\right)}{\left(\frac{\beta}{r}-\frac{\beta^3}{6r^3}\right)\left(\frac{\gamma}{r}-\frac{\gamma^3}{6r^3}\right)} \quad .....(43)$$

In the above expression the expansions are only carried as far as the terms $\alpha^4, \beta^4, \gamma^4$ for the reasons already stated above.

After some simplification and neglecting terms above the 4th order, the expression(43) on the previous page becomes:-

$$\cos A = \frac{1}{2\beta\gamma}\left[\beta^2+\gamma^2-\alpha^2+\frac{\alpha^4+\beta^4+\gamma^4-2\alpha^2\beta^2-2\alpha^2\gamma^2-2\beta^2\gamma^2}{12r^2}\right] \quad \ldots\ldots\ldots(44)$$

Let A',B',C' be the angles of a plane triangle, whose sides are α, β, γ.

$$\sin^2 A' = 1-\cos^2 A'$$

$$= 1-\left(\frac{\beta^2+\gamma^2-\alpha^2}{2\beta\gamma}\right)^2$$

$$= \frac{2\alpha^2\beta^2+2\alpha^2\gamma^2+2\beta^2\gamma^2-\alpha^4-\beta^4-\gamma^4}{4\beta^2\gamma^2}$$

$$\cos A = \cos A' - \frac{\beta\gamma}{6r^2}\sin^2 A'$$

Now A' differs from A by only a small quantity θ(say).

i.e. $A = A'+\theta$

$\cos A = \cos A' \cos\theta - \sin A' \sin\theta$

As θ is a very small angle $\cos\theta = 1$, $\sin\theta = \theta$

Therefore

$\cos A = \cos A' - \sin A'\,\theta$

or $\theta = \frac{\beta\gamma}{6r^2}\sin A'$

$= \frac{S}{3r^2}$, where S area of the triangle A'B'C'.

Hence

$A = A' + \frac{S}{3r^2}$ and similarly $B = B' + \frac{S}{3r^2}$ and $C = C' + \frac{S}{3r^2}$

Adding $A+B+C-(A'+B'+C') = \frac{S}{r^2} = \frac{\text{Area of } \triangle ABC}{r^2}$

$= A+B+C-\Pi$ or the Spherical excess

<u>Hence we have Legendre's theorem</u> (vide (41), p.page)

viz:- that if we have given the side c of a spherical triangle and we form a plane triangle with the side $c' = c$ and subtract $\frac{1}{3}$rd of the spherical excess from each of the 3 spherical angles A ,B , C , to obtain the angles A',B', C', of a corresponding plane triangle, and solve the latter, we obtain $a'=a$, $b'=b$. ....(45)

If the observed angles were free from error, we would only have to add ABC together and subtract 180° to obtain the spherical excess. As however this is not the case, we must obtain it by other means.

The spherical excess $= \frac{S}{r^2} = \frac{1}{2r^2}b'c' \sin A'$

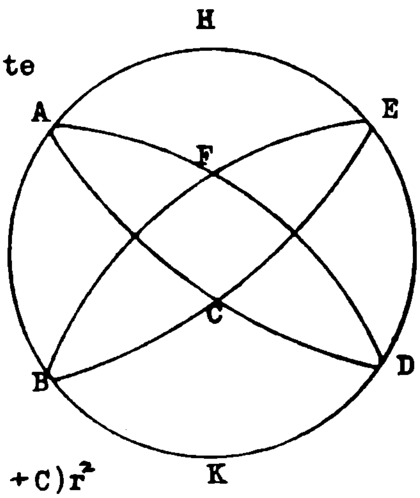
$$= \frac{1}{2r^2}c'^2 \frac{\sin A' \sin B'}{\sin C'} \quad \ldots\ldots(46)$$

In this formula it is sufficient to put $A=A'$, $B=B'$, $C=C'$

In the figure ACDF and BCEF are great circles

It can be seen by symmetry that the triangle ABC = triangle DEF, at opposite sides of the diameter.

Fig 9



Triangle ABC + BCDK = lune A

$= 2Ar^2$

Triangle ABC + ACEH = lune B

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Triangles ABC + CDE = lune C

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2 (triangle ABC) + hemisphere = $2(A + B + C)r^2$

2(triangle ABC) = $2(A + B + C - \pi)r^2$

Triangle ABC = $(A + B + C - \pi)r^2$

$A + B + C - \pi$ is called the Spherical Excess ......(41)

$$\frac{\text{Area of Triangle ABC}}{\text{Area of hemisphere}} = \frac{A + B + C - \pi}{2\pi} = \frac{\text{Sph Excess}}{4 \text{ rt } \angle s} \quad \text{....(42)}$$

Very few sides of a triangulation exceed 100 miles. The circular measure of this is, (taking the earth's radius roughly as 4000 miles)

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Let α, β, γ be the lengths of the sides a, b, c, of the triangle ABC

$$\cos A = \frac{\cos a - \cos b \cos c}{\sin b \sin c}$$

Expanding the right hand side by the expansions for sine and cosine in circular measure we get:-

$$\cos A = \frac{\left(1 - \frac{\alpha^2}{2r^2} + \frac{\alpha^4}{24r^4}\right) - \left(1 - \frac{\beta^2}{2r^2} + \frac{\beta^4}{24r^4}\right)\left(1 - \frac{\gamma^2}{2r^2} + \frac{\gamma^4}{24r^4}\right)}{\left(\frac{\beta}{r} - \frac{\beta^3}{6r^3}\right)\left(\frac{\gamma}{r} - \frac{\gamma^3}{6r^3}\right)} \quad \text{.....(43)}$$

In the above expression the expansions are only carried as far as the terms $\alpha^4, \beta^4, \gamma^4$ for the reasons already stated above.

After some simplification and neglecting terms above the 4th order, the expression(43) on the previous page becomes:-

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Let A',B',C' be the angles of a plane triangle, whose sides are α, β, γ.

$$\sin^2 A' = 1-\cos^2 A'$$
$$= 1-\left(\frac{\beta^2+\gamma^2-\alpha^2}{2\beta\gamma}\right)^2$$
$$= \frac{2\alpha^2\beta^2+2\alpha^2\gamma^2+2\beta^2\gamma^2-\alpha^4-\beta^4-\gamma^4}{4\beta^2\gamma^2}$$

$$\cos A = \cos A' - \frac{\beta\gamma}{6r^2}\sin^2 A'$$

Now A' differs from A by only a small quantity θ(say).

i.e. $A = A'+\theta$

$\cos A = \cos A' \cos\theta - \sin A' \sin\theta$

As θ is a very small angle $\cos\theta = 1$, $\sin\theta = \theta$

Therefore

$$\cos A = \cos A' - \sin A'\,\theta$$
$$\text{or } \theta = \frac{\beta\gamma}{6r^2}\sin A'$$
$$= \frac{S}{3r^2}$$, where S area of the triangle A'B'C'.

Hence

$$A = A' + \frac{S}{3r^2} \text{ and similarly } B = B' + \frac{S}{3r^2} \text{ and } C = C' + \frac{S}{3r^2}$$

Adding $A+B+C-(A'+B'+C') = \frac{S}{r^2} = \frac{\text{Area of } \triangle ABC}{r^2}$

$= A+B+C-\pi$ or the Spherical excess

<u>Hence we have Legendre's theorem</u> (vide (41), p.page)

viz:- that if we have given the side c of a spherical triangle and we form a plane triangle with the side $c' = c$ and subtract $\frac{1}{3}$rd of the spherical excess from each of the 3 spherical angles A ,B , C , to obtain the angles A',B', C', of a corresponding plane triangle, and solve the latter, we obtain $a'=a$, $b'=b$. ....(45)

If the observed angles were free from error, we would only have to add A,B,C together and subtract 180° to obtain the spherical excess. As however this is not the case, we must obtain it by other means.

The spherical excess $= \frac{S}{r^2} = \frac{1}{2r^2}b'c' \sin A'$

$$= \frac{1}{2r^2}c'^2 \frac{\sin A' \sin B'}{\sin C'} \ldots\ldots (46)$$

In this formula it is sufficient to put A=A', B=B', C=C'

Also to convert the angle, which is in circular measure to seconds, it has to be divided by the circular measure of 1 second or approx by the sine of 1 second, or mutiplied by cosec 1".

Thus finally the Spherical excess $= \dfrac{c^2 \sin A \sin B}{\sin C} \times \dfrac{\text{cosec } 1''}{2r^2}$ ..(47)

other formulae are

Spherical excess $=$ bc sin A x $\dfrac{\text{cosec } 1''}{2r^2}$ ....(48)

$=$ area of $\triangle$ x $\dfrac{\text{cosec } 1''}{r^2}$ ....(49)

SUMMARY of Important Formulae in Spherical Trigonometry(vide p.pages)

1. ab = AB cos(lat)

2. Rt ∠ Sph $\triangle$s. Write for ∠ or Hyp, (90°-∠) or (90°-Hyp) omitting the rt ∠. Then Napier's Rules are:-

   sin(any part) = product cosines opp parts

   = " " tangents adj "

3 Ordinary Spherical $\triangle$s.

$$\frac{\sin A}{\sin a} = \frac{\sin B}{\sin b} = \frac{\sin C}{\sin c} \quad ..... (a)$$

$$\cos a = \cos b \cos c + \sin b \sin c \cos A$$
$$\cos b = \cos a \cos c + \sin a \sin c \cos B \quad .....(b)$$
$$\cos c = \cos a \cos b + \sin a \sin b \cos C$$

$$\cos A = -\cos B \cos C + \sin B \sin C \cos a$$
$$\cos B = -\cos A \cos C + \sin A \sin C \cos b \quad .....(c)$$
$$\cos C = -\cos A \cos B + \sin A \sin B \cos c$$

Taking any four consecutive parts

cos(middle side)x cos(middle ∠) = sin(middle side)x cot(other side) − sin(middle ∠) cot(other ∠) .....(d)

$$\sin\frac{A}{2} = \sqrt{\frac{\sin(s-b)\sin(s-c)}{\sin b \sin c}} \qquad \tan\frac{A}{2} = \sqrt{\frac{\sin(s-b)\sin(s-c)}{\sin s \sin(s-a)}} \quad ...(e)$$

$$\cos\frac{A}{2} = \sqrt{\frac{\sin(s-a)\sin s}{\sin b \sin c}}$$

$$\tan\left(\frac{A+B}{2}\right) = \frac{\cos\left(\frac{a-b}{2}\right)}{\cos\left(\frac{a+b}{2}\right)} \cot\frac{C}{2} \quad ..(f)$$

$$\tan\left(\frac{A-B}{2}\right) = \frac{\sin\left(\frac{a-b}{2}\right)}{\sin\left(\frac{a+b}{2}\right)} \cot\frac{C}{2} \quad ..(g)$$

$$\tan\left(\frac{a+b}{2}\right) = \frac{\cos\left(\frac{A-B}{2}\right)}{\cos\left(\frac{A+B}{2}\right)} \tan\frac{c}{2} \quad ...(h)$$

$$\tan\left(\frac{a-b}{2}\right) = \frac{\sin\left(\frac{A-B}{2}\right)}{\sin\left(\frac{A+B}{2}\right)} \tan\frac{c}{2} \quad .....(i)$$

$$\sin\frac{a}{2} = \sqrt{\frac{-\cos S \cos(S-A)}{\sin B \sin C}} \quad (j)$$

$$\cos\frac{a}{2} = \sqrt{\frac{\cos(S-B)\cos(S-C)}{\sin B \sin C}} \quad .(k)$$

$$\tan\frac{a}{2} = \sqrt{\frac{-\cos S \cos(S-A)}{\cos(S-B)\cos(S-C)}} \quad .(l)$$

SUMMARY of Important Formulae in Spherical Trigonometry. (continued)

Area of lune = $2r^2\theta$ ....(j)

Spherical Excess = $A+B+C-\pi = \frac{S}{r^2}$......(k)

If we have a sph△ ABC and we make a plane△ A'B'C' sides a', b', c', so that $A' = A - \frac{1}{3}$ rd Sph Excess , $B' = B - \frac{1}{3}$rd Sph Excess $C' = C - \frac{1}{3}$rd Sph Excess, then $a = a'$, $b = b'$, $c = c'$......(l)

The Spherical Excess in seconds is given by:-

$$\frac{c^2 \sin A \sin B}{\sin C} \times \frac{\operatorname{cosec} 1''}{2r^2} \quad ....(m)$$

$$\underline{\text{or}} \quad bc \sin A \times \frac{\operatorname{cosec} 1''}{2r^2} \quad ......(n)$$

$$\underline{\text{or}} \text{ area of } \triangle \times \frac{\operatorname{cosec} 1''}{r^2} \quad ....(o)$$

The formulae(25) to (39) on pages 9 to 13 should also be remembered, which are modifications of formulae (10),(11), and (15), of pages 6 and 7,(i.e.- of (b),(c) and (d) in the summary previous.)

Convergency, as applied to a traverse, plotted on the Cassini Projection

To carry out a system of rectangular coordinates based on the assumption that the earth up to a limit of about 2 degrees either side of a central origin is a plane, as is done on the Cassini projection, we have to treat the meridians at successive stations of a traverse as all parallel to one another, and the lines of departure East and West all as perpendicular to the meridians.

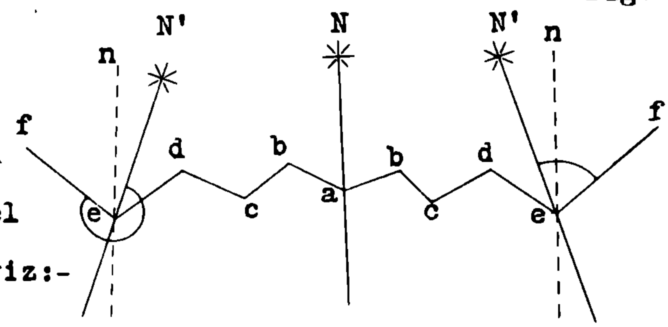
The true bearing at the origin of the survey is astronomically determined and adopted as the working bearing in our traverse calculations. It is clear however that the true bearing astronomically observed at other points E or W of the origin cannot be introduced into our calculations, since they are observed from converged meridians. The amount of convergency must therefore be eliminated from an observed bearing before we can use it. The observed bearing, thus divested of convergency is called the reduced bearing. The bearing of any line deduced from the bearing at the origin through successive stations by the observed angles of the traverse circuit is called the deduced bearing. If the observed angles of the traverse were all absolutely correct, the deduced and reduced bearings would be exactly the same, but we cannot assume the angular work to be faultless, nor can we tell in which particular part of the traverse nor to what extent corrections to the angles are needed without taking azimuth observations at certain intervals.

Azimuth observations are necessary about every 5 to 10 miles East or West of the origin, according as the ordinary length of the legs of the traverse are short or long. The method of applying the correction for convergency is as follows:-

Let a be the starting point and origin of a traverse and aN be the true meridian at this point.

The survey proceeds E or W, ab, bc , cd , de to e

Fig. 10



At e, we determine the true meridian eN' by an azimuth observation, which is inclined to the parallel to the original meridian,viz:- to en.

The measure of the convergency is therefore the angle N'en

Now the true forward bearing of the next station f is the angle N'ef. Therefore the reduced bearing is $\angle$N'ef $\pm\angle$N'en,according as e is $\frac{W}{E}$ of the station of origin $= \angle$ nef

But the deduced bearing from the computation form of the traverse is also the angle nef. If therefore the reduced and deduced bearing do not agree, the error,or difference between the two,should be distributed back among the number of stations a,b,c,d,e,less one, in the bearings of the lines ab,bc,cd,de, plus or minus , as the case may be.

From the above it is clear that the difference between the reciproc -al true bearings(Azimuths) between two stations is the measure of the Convergency, which we must apply to a traverse, based on the Cassini system of Rectangular coordinates. It is necessary therefore to determine a formula for its value at any given distance from the origin. It may be noted however that this formula only applies to the Cassini system,and not to other projections such as the Lambert Conical Orthomorphic in which the formulae differ.

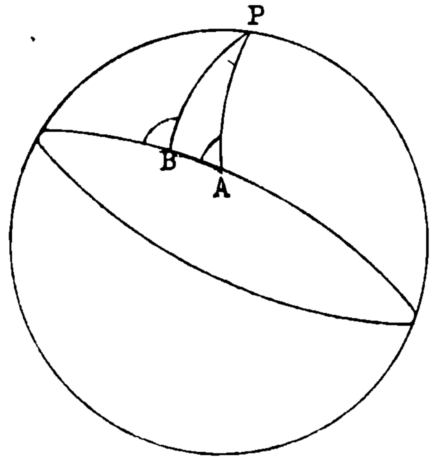
Let A and B be two points on the earth's surface. The straight line between them, being the shortest distance between them, is part of a great circle.

The Azimuth of B from A = $\angle A$

The reciprocal Azimuth of A from B = $\angle\ 180^\circ - B$, measured in the sa sense. The convergency is thus $180^\circ - A - B$.

Now the meridians through A and B of the Sph $\triangle$ PAB converge and intersect at the pole P.

Fig. 1



$\angle P = \triangle L$, the diffce of longitude

$AP = 90 - \lambda_1$, λ_1 being latitude of A

$BP = 90 - \lambda_2$, λ_2 being latitude of B

$$\tan\frac{A+B}{2} = \frac{\cos\frac{1}{2}(a-b)}{\cos\frac{1}{2}(a+b)}\cot\frac{P}{2}$$

$$= \frac{\cos\frac{1}{2}(90^\circ - \lambda_1 - 90^\circ + \lambda_2)}{\cos\frac{1}{2}(90^\circ - \lambda_1 + 90^\circ - \lambda_2)}\cos\frac{\triangle L}{2}$$

or

$$\cot\left(90 - \frac{A+B}{2}\right) = \frac{\cos\frac{1}{2}(\lambda_1 - \lambda_2)}{\sin\frac{1}{2}(\lambda_1 + \lambda_2)}\cot\frac{\triangle L}{2}$$

or

$$\tan\frac{c'}{2} = \frac{\sin\frac{1}{2}(\lambda_1 + \lambda_2)}{\cos\frac{1}{2}(\lambda_1 - \lambda_2)}\tan\frac{\triangle L}{2}$$

,where c' is the convergency.

When AB is small, compared with the radius of the earth, we may substitute for the tangents of the angles their value in radian measure, so that

$$c' = \triangle L\,\frac{\sin\frac{\lambda_1 + \lambda_2}{2}}{\cos\frac{\lambda_1 - \lambda_2}{2}} \quad \ldots\ldots\ldots\ldots(50)$$

When A and B lie on the same parallel, this reduces to

$$c' = \triangle L \sin\lambda \quad \ldots\ldots\ldots\ldots(51)$$

When A and B lie on the same meridian $\triangle L = 0$ and $c' = 0$

When A and B lie on the equator $\lambda_1 = \lambda_2 = 0$ and $c' = 0$

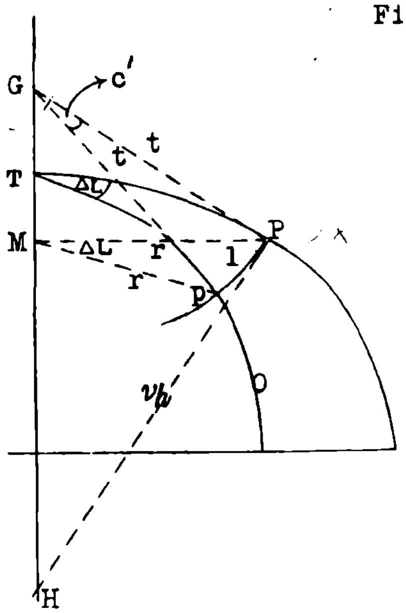


Fig. 12

Another direct proof of the formula (51) is given below. In the figure TO is the initial meridian of the survey, P is a point on the spheroid, TP its meridian, Pp is the parallel of latitude through P: GP and Gp $= t$ are tangents and PH$=\nu_h$ is the normal, respectively in the latitude of P; PM and pM $= r$ are radii of the arc Pp, and the angle PMp$=\triangle L$. Let the angle PGp$= c'$, the length of the arc Pp$= l$ and the latitude of P $=\lambda_h$

Then in $\triangle$ MPH $\quad r=\nu_h \cos\lambda_h$

and " " GPH $\quad t=\nu_h \cot\lambda_h$

Now about the centre M, we have $l=r\,\triangle L \sin 1''$

and " " " " " " G " " " " $l=tc'\sin 1''$

Substituting for r and t from above

$$c'\nu_h \cot\lambda_h = \triangle L\, \nu_h \cos\lambda_h$$

$$\text{or } c' \text{ (the convergency)} = \triangle L \sin\lambda_h \quad \ldots.(51)$$

When the convergency at two points not in the same latitude is required, the convergency at the mean latitude or $\triangle L \sin\frac{\lambda_1+\lambda_2}{2}$ is usually taken. This differs but little from formula (50) on the previous page, as the difference of latitude is usually very small, and $\cos\frac{\lambda_1-\lambda_2}{2}$ practically $= 1$

The convergency is sometimes given in a slightly different form. As the x coordinate has the value $\triangle L\nu\cos\lambda$ on the Cassini system of coordinates, we have $\frac{(x\text{ coordinate})x \tan\lambda \operatorname{cosec} 1''}{\nu}$ = convergency in seconds ...(52)

Astronomical Notation and Symbols used in this pamphlet.
(in conformity with those adopted in the Survey Profl forms

λ Latitude

γ Colatitude

L Longitude(in arc) - l Longitude(in time)

δ Declination

$\triangle$ North Polar Distance or N.P.D.

h Altitude

ζ_0 Zenith Distance (observed). ζ Corrected Z.D.

t Hour Angle

A Azimuth

R.A. Right Ascension

R.M. Referring Mark

N.A. Nautical Almanac

A.E. American Ephemeris

H Barometer

T Temperature

"r" Refraction

"p" Parallax

S.T. Sidereal Time

L.S.T. Local Sidereal Time

G.S.T. Greenwich Sidereal Time

L.A.T. Local Apparent Time

G.A.T. Greenwich Apparent Time

L.M.T. Local Mean Time

G.M.T. Greenwich Mean Time

L.A.N. Local Apparent Noon

G.A.N. Greenwich Apparent Noon

G.A.M. (1925) Greenwich Apparent Midnight (vide para 16)

L.M.N. Local Mean Noon

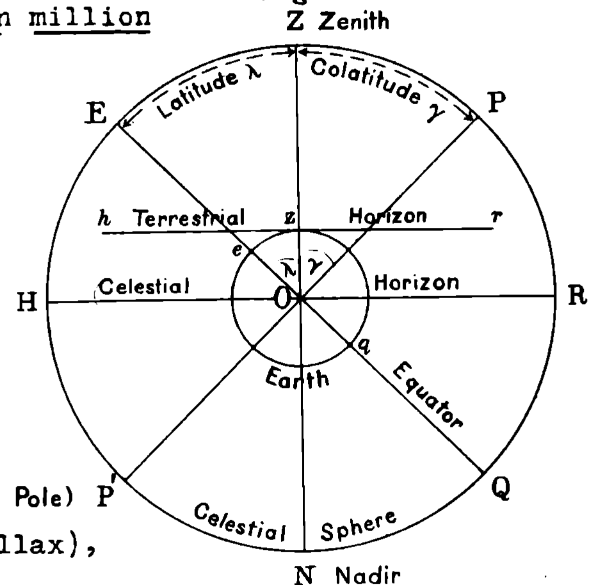
G.M.N. Greenwich Mean Noon

G.M.M. (1925) Greenwich Mean Midnight (vide para 16)

NOTES ON ASTRONOMY ✓

1. All celestial bodies except the sun, moon, planets etc., of the solar system, are at such vast distances (e.g.:-α_2 Centauri, 20 <u>million</u> <u>million</u> miles), that,even when viewed from opposite points of the earth's orbit, the apparent change in their directions never exceeds 1" of arc. The change in direction in viewing a fixed star therefore, due to an observer being at the earth's surface and not at its centre (geocentric parallax), is entirely negligible. Wherefore,

Fig 1



The plane of the paper is supposed to coincide with the plane of the meridian

although to an observer at a station of observation on the earth's surface the heavens appear as a vast concave sphere drawn round his actual station as centre, for all investigations regarding stars we can suppose the sphere drawn with its centre at the centre of the earth and not at the station of observation. When we have to deal with the sun, moon, or planets, however, which are closer to the earth, we have to introduce a correction for (geocentric) parallax, which will be explained hereafter. The points in which the earth's axis P P' cuts the celestial sphere are called the <u>Poles</u> of the celestial sphere. (vide Fig 1.) These are fixed points on the celestial sphere. If we draw a line through the centre of the sphere O to any point z on the earth's surface, and produce it to cut the celestial sphere in Z, this point, vertically above the place, is called the Zenith. If produced in the other direction, it cuts the celestial sphere in the point vertically beneath, or <u>Nadir</u>, N. Thus if we can locate a place on the earth's surface, we know the position of its zenith, and vice versa. In future therefore the position of a place will be denoted by the point Z.

2. The earth's Equator"eq"is the great circle in which a plane through the centre of the earth, perpendicular to the axis, cuts the earth's surface. This plane produced cuts the celestial sphere in the Celestial Equator or Equinoctial EQ. The great circles through the celestial poles, which therefore cut the celestial equator at right angles, are called Celestial Meridians. Now we know that the angle, which the part of a terrestial meridian "ze", intercepted between a place z and the terrestial equator, subtends at earth's centre O, is the Latitude λ of the place. Similarly therefore the Latitude of a place is the angle subtended at the centre by the arc of the Celestial Meridian ZE, intercepted between the Zenith and the Celestial Equator, and the Colatitude γ is the angle subtended at the centre by the arc of the Celestial Meridian ZP, intercepted between the Zenith and the Pole. ($\gamma = 90^\circ$ – latitude λ).

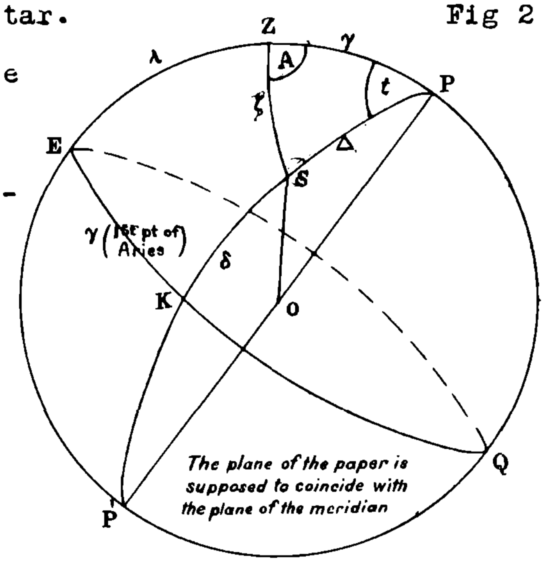
3. If a plane be drawn through the centre O of the earth perpendicular to OZ, it cuts the celestial sphere in a great circle called the Celestial Horizon, HR. This is really parallel to the terrestial horizon "hr", but, as the size of the earth is so small compared with the distance of the stars, they may be considered identical for our purpose.

The points in which the observer's meridian cuts the horizon are the N & S points.

Great circles through the Zenith are of course perpendicular to the horizon and are called Verticals. The vertical, whose plane is perpendicular to the plane of the meridian, is called the Prime Vertical and meets the horizon in the E & W points.

4. Draw a line from the earth's centre to a star. Let it cut the celestial sphere in S. SZ is the <u>Zenith Distance</u> ζ, and SP the <u>North Polar Distance</u> Δ, of the star.

Fig 2



Just as the position of a place on the surface of the earth is determined if we know its latitude and longitude, so a star's position is determined if we know the distance SK measured along the meridian between the star and the equator, and also the distance ΥK measured from a fixed point Υ up to K along the equator.

ΥK is called the star's <u>Right Ascension</u>, or R.A.

SK .... <u>Declination</u>, δ. (vide Fig 2)

Υ is called the First Point of Aries and its position will be determined later on, (vide para 9)

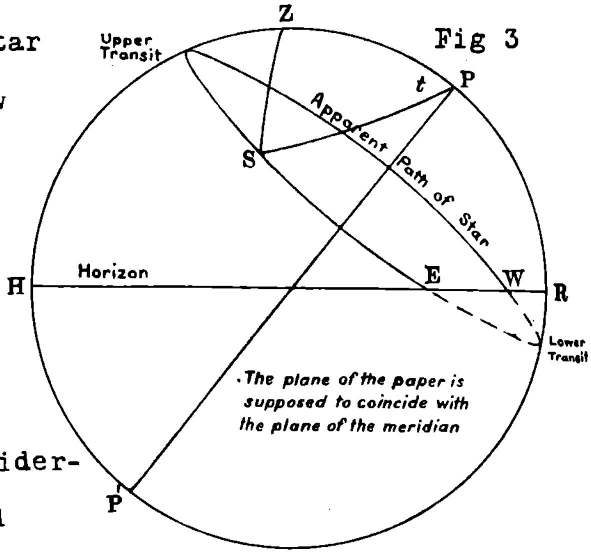
The Declination, δ = <u>90^o - North Polar Distance</u>, Δ.

The $\angle$PZS is the star's <u>Azimuth</u>, A, measured from north.

5. The earth rotates on its axis once in 24 hours, but we are not cognisant of the fact except indirectly. What we do see is that the sun and stars travel round the earth once in 24 hours, rising in the East and setting in the West. This is just what we would observe if the stars were fixed and the earth turned on its axis from W to E. It is much more likely that the earth rotates than that the rest of the universe turns round it, and it can be proved that it does. All the same it is more convenient to suppose that the stars, etc, travel round the earth once in 24 hours.

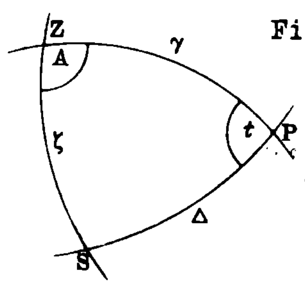
6. A star whose N.P.D. is 90° will therefore appear to travel on the equator and any other star will travel on a small circle parallel to the equator. As a star cannot be seen, when it is below the horizon, it will appear to rise at one of the points, E, where its small circle cuts the horizon, and set at W, the other. As a star does a complete revolution or 360° in 24 Sidereal hrs., it does 15° per hour, and the ∠ SPZ, reduced to hours at the rate of 15° per hour, will give the time after which the star will be on the meridian. The ∠ SPZ is therefore called the Hour Angle t.

Fig 3



7. Astronomical triangle. It is thus clear that a star (or other heavenly body) at any moment forms a spherical triangle with the zenith and elevated pole (vide Fig 4), the sides being arcs of the meridian ZP, the vertical circle ZS and the declination circle PS.

Fig 4



The side PZ = Colatitude γ (or $90^\circ - \lambda$, the Latitude)

the side PS = N.P.D. Δ (or $90^\circ - \delta$, the Declination)

the side ZS = Zenith distance ζ or $90^\circ - h$, the Altitude)

the side PZ is generally known approximately and can be found accurately, as explained afterwards.

the side PS is obtained from the value of Declination given in the Nautical Almanac.

the side ZS is obtained by observation with theodolite or other instrument, if required.

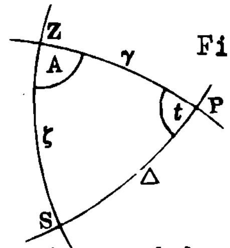


Fig 4

The angle ZPS is the **Hour Angle** t

The angle PZS is the Azimuth Angle A

The angle ZSP is the Parallactic Angle.

Given any 3 of these parts, the others can be deduced by formulae of spherical trigonometry.

We thus are able to determine Latitude, Time (chronometer error) and Azimuth, which are the problems most frequently occurring in Field Astronomy.

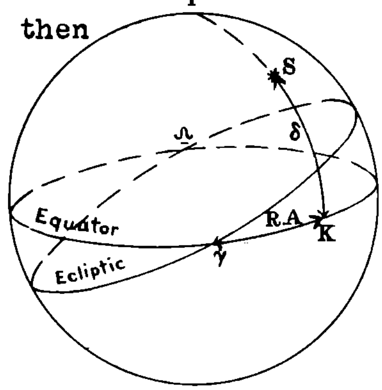
8. Besides the diurnal motion round its axis the earth has another motion. It revolves round the sun from E to W once a year. We are not aware of this; what we do see is that the sun moves among the stars from W to E. For supposing the sun had no apparent motion, then noon would always be the same, and the time between noon and the rising of any star would always be the same. But a star rises earlier every night, which means that the interval is getting smaller every day, or that the sun is getting further East each day.

We shall speak of the sun moving round the earth, or the earth moving round the sun, just as it suits us. The earth is not always equidistant from the sun. It is nearer in winter and further away in summer. In fact the earth moves round the sun in an ellipse with the sun in one focus, and in consequence of this, equal areas must be described in equal times, the earth moving faster at some times than at others.

9. The plane of the earth's orbit, as this ellipse is called, or, which is the same thing, the plane of the sun's apparent motion is not parallel to the equator. It cuts it at an angle of about $23\frac{1}{2}^{\circ}$, so that the great circle in which this plane cuts the celestial sphere, and which is called the **Ecliptic**, is inclined

to the equator at an ∠ of about 23½°. This is called the Obliquity of the Ecliptic. The sun thus appears to move in the ecliptic and the point where it crosses the equator, when ascending, is called the 1st Point of Aries and is denoted by ♈, and, when descending, by ☊.

Fig 5



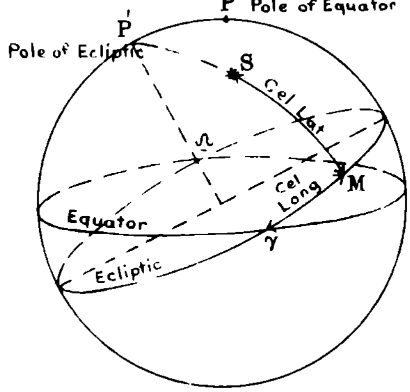
10. A star's Right Ascension or R.A. is then the arc ♈K of the equator measured from ♈ eastward to the point K, where the star's declination circle SK cuts the equator. Right Ascensions are reckoned from 0 to 360°, or, which is the same thing, from 0 to 24 hours.

11. If we take the pole of the ecliptic and draw a great circle through it and through a star "S", meeting the ecliptic in M, say, then the star's position will also be known, if we know ♈M and MS :-

♈M is the star's Celestial Longitude
MS .... Celestial Latitude

These coordinates are not wanted for the simple calculations we will consider.

Fig 6



12. The 1st Point of Aries is not actually a fixed point. It has a retrograde motion of 50.22" seconds annually, moving, as it were, to meet the earth. When the sun is at ♈ and ☊, it is on the equator and therefore night & day are of the same length. These two positions are called the Equinoxes. The motion of ♈ along the ecliptic is called the Precession of the Equinoxes.

13. We are now in a position to determine various measures of time. As the earth turns on its axis, the obvious unit of time is the duration of one revolution, or, as we see it, it is the time that elapses from the transit of a heavenly body over the meridian till its next transit.

If a star be the object selected, the interval will be a Sidereal day. As a matter of fact, for the purpose of measuring Sidereal time, it is not a star which is selected, but the 1st point of Aries. A Sidereal day therefore begins when ♈ is on the meridian. A correct Sidereal clock should then mark 0/h 0/m 0/s, and, at any other instant, the sidereal time will be the hour angle of ♈ reckoned westward from 0 to 24 hours.

14. A Solar day is the interval between 2 successive transits of the sun's centre over the meridian, but we saw that the sun moved eastward about 1^{o} per diem, so that the earth will have to move through 361^{o}, to complete a Solar day, which will therefore be some 4 minutes longer than a Sidereal day.

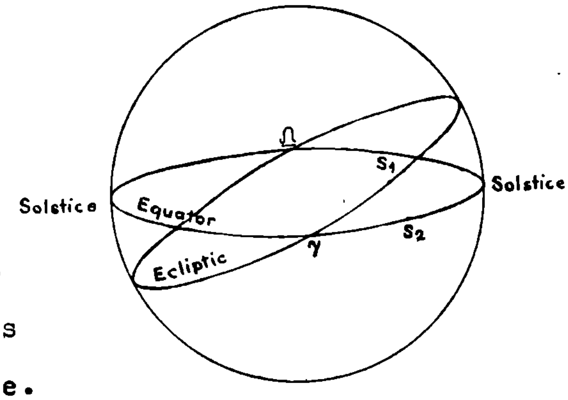
The solar time at any instant is the hour angle of the sun's centre, reckoned westward from 0 to 24 h. This is called Apparent Solar Time and is the time indicated by a Sundial. If the sun's motion in right ascension were uniform, Solar days would all be equal, but this is not the case. In the first place the sun's motion in his own orbit is not uniform, and secondly, even if it were, the corresponding motion in right ascension would not be uniform, owing to the inclination of its orbit to the equator.

15. We can however obtain a uniform measure of time, depending on the mean or average motion of the sun, in the following way:- Let an imaginary Mean Sun, S_1, move in the ecliptic with the true

sun's mean or average angular velocity, and let it coincide with the sun when it is nearest the earth, and therefore also, when it is furthest away.

Let a second Mean Sun S_2 move in the equator, so that ΥS_2 always $= \Upsilon S_1$, then S_2 is the Mean Sun that we require.

Fig 7



Mean Noon is the instant when the Mean Sun is on the meridian, and Mean Time is the hour angle of the Mean Sun, reckoned westward either from 0 to 24 hours, or in 2 twelve hourly periods. Astronomical Time is always on the 24 hour, and not on the 12 hour system, commonly used for civil purposes.

16. Prior to 1925 the Nautical Almanac, American Ephemeris, etc., used to reckon their dates in Astronomical Time, starting each day at 12 noon, but, as for civil purposes it has been found more convenient to begin the day at midnight, the Almanacs have carried out this change, commencing with those issued for 1925, so as to avoid the confusion between Civil and Astronomical dates, caused by the old system.

17. Suppose for a moment we neglect the fact that the true sun moves at different rates in his orbit. Under this supposition the true sun's Right Ascension (owing to its moving in the ecliptic) will be sometimes greater and sometimes less than that of the Mean Sun. They would coincide at the Equinoxes and 90°

from them at points called the Solstices. Apparent Time will therefore be sometimes ahead of, and sometimes behind, Mean Time and the difference between them is called the Equation of Time. The Equation of time is thus the value, expressed in time of the angle between the true and mean suns. The fact that the true sun travels at varying rates in his orbit alters the amount of the Equation of Time, and also the dates on which it vanishes, but not the number of times (viz:- 4) on which it vanishes. The Equation of Time is given for every day in the year in the Nautical Almanac (first 2 pages of each month) and we need not concern ourselves with its value beyond noting that it varies from 0 to about ± 15m.

18. The next unit of time is the year. A year is the period of the earth's revolution about the sun from some determinate position back to the same. If the starting point be a star, the interval is called a Sidereal year. If we start from the 1st point of Aries, which has a retrograde motion of 50.22" per annum, moving as it were to meet the earth, the period will not be so long. This is called the Tropical year, and, as it determines the commencement of the seasons and all the important phenomena of vegetation and life, it is the unit marked out by nature for the use of man.

19. From observations separated by a long interval it has been found to consist of 365.242216 Mean Solar days. As this is an awkward number, the ordinary civil year is made to consist of an exact number of days either 365 or 366.

Now 4 Tropical years = 4 years of 365 days + .968864 day
= 3 years of 365 days + one year of 366 days − .031136 day,

so that in the ordinary way of having every fourth year a leap year, we should get an error of .031136 days in 4 years, or of

3.1136 days in 400 years. Hence the further correction of not counting as a leap year any century, unless its number is divisible by 400. (Thus 1900 was not a leap year, but 2000 will be). The error then in the calendar, as at present reckoned, is .1136 day in 400 years.

20. Reduction and Conversion of Time:- As the earth turns uniformly on its axis, one meridian after another is brought opposite the sun and different places have their noons in succession according to their longitude. The Solar Time at a given place, being the angle made by the sun's declination circle with the meridian at that place, it follows that the difference between the Solar Times at 2 different places at the same instant will be exactly the angle between the meridians of the 2 places (i.e. their difference in longitude). The same will be true of their Mean Solar, or of their Sidereal, times, and generally the difference of Longitude will be equal to the difference of Hour Angles of any (the same) celestial point at the same instant. Therefore to find the time at any meridian, corresponding to a given time at some other meridian, we must convert the Longitude into time at the rate of 15^{o} per hour, and add to, or subtract from, the given time.

21. Bear in mind that the earth turns from W to E and the heavenly bodies travel from E to W apparently, so that the more easterly meridian will have its noon first, and therefore the more advanced time. e.g. The Longitude of Dehra is $78^{o}.5'.42''$ E, what is Mean Time at Dehra, when it is Mean Noon at Greenwich? We must divide $78^{o}.5'.42''$ by 15. The result is 5h. 12m. 22.8", and, as Dehra is E of Greenwich, it is 5h. 12m. 22.8s, Mean Time at Dehra, when it is Mean Noon at Greenwich.

Conversely, if the difference of time were given, and we wanted the difference in longitude, we should multiply the difference of time, 5h. 12m. 22.8s, by 15, and get the result $78^{\circ}\ 5'\ 42''$.

22. The interval between 2 successive returns of ♈ to the same meridian is a Sidereal day and that between 2 successive returns of the mean sun is a Mean Solar day. Now the sun completes an apparent revolution round the earth in a tropical year consisting of 365.242216 mean solar days.

$$\frac{\text{Daily motion}}{360^{\circ}} = \frac{1 \text{ day}}{365.242216} \quad \text{Daily motion} = 59'\ 8''.33$$

The length of the Mean Solar day therefore differs from the length of the Sidereal, because, when the mean sun in its diurnal motion returns to the meridian, it is $59'\ 8''.33$ advanced in R.A. eastward, i.e:-an arc of the equator of $360^{\circ}\ 59'\ 8.33''$ passes the meridian in a Mean Solar day, while one of only 360° passes in a Sidereal day.

$$\frac{1 \text{ Mean Solar day}}{1 \text{ Sidereal day}} = \frac{360^{\circ}\ 59'\ 8.33''}{360^{\circ}} = \frac{24\text{h. } 3\text{m. } 56.555\text{s.}}{24 \text{ hr.}} = \frac{1\text{h.}9.8565\text{s}}{1 \text{ hr.}}$$

23. It now remains to show how to convert Sidereal into Mean Solar time, and vice versa. The earth turns on its axis, so that the sun and ♈ appear to revolve round the earth. Every time the mean sun crosses the meridian is a Mean Solar day, and every time ♈ crosses the meridian is a Sidereal day. Now the mean sun advances among the stars in the same direction as the earth revolves, viz:-, from W to E. Therefore the mean sun will be a little later crossing the meridian each day, and, finally, as the mean sun goes once completely round the earth in a tropical year, the mean sun will be a whole day later crossing the meridian than ♈ is. Thus there will be one less Mean Solar day in a tropical year than there are Sidereal days.

Number of Mean Solar days in a Tropical year = 365.242216
.... Sidereal .... = 366.242216

Therefore 365.242216 Mean Solar days = 366.242216 Sidereal days.

1 Mean Solar day = 1 + .00273791 Sidereal days
1 Sidereal day = 1 - .00273043 Mean Solar days.

Tables for the conversion are given in the Nautical Almanac, American Ephemeris, Chamber's Log Tables and Auxiliary Tables, Part III, Tables 22, 23 Sur.

24. In order to get a clear conception of the various kinds of time in use, a few examples should be worked such as the following:- Near the Walker Observatory, Dehra Dun, longitude 78° 3' 15", the Local Apparent Time on 1st August 1929 is 20 hours, what is the Local Mean Time and what is the Standard Time?

L 78° 3' 15" E = $5^h\ 12^m\ 13^s$ E in time

G.A.T. at 20 hrs. = $20^h - 5^h\ 12^m\ 13^s = 14^h\ 47^m\ 47^s$.

The Equation of Time at $2^h\ 47^m\ 47^s$ after noon at Greenwich is required

Equation of Time from p 1 of August 1929 N.A.

at Apparent Noon	= + $6^m\ 11.70^s$
change in 2.8^h approx. at .139	= − .39
Equation of Time required	= + 6 11.31

L.A.T.	= 20.0.0
Equation of Time	= + 6 11.31
L.M.T. required	= 20 6 11.31

	h m s	
Difference between longitude of Dehra	5 12 13	
and longitude for standard meridian.	5 30 0	+17 47
Standard time required		20 23 58.31

25. Now in astronomical problems we often require to know the **Sidereal Time of Mean Noon** i.e. the Right Ascension of the Mean

Sun, when in the meridian. The Sidereal Time of Mean Noon for Greenwich is given for every day in the year in the <u>**Nautical Almanac**</u> p II of each month, but we want the Sidereal Time of Local Mean Noon, at Dehra Dun, say. Now the only difference between the Sidereal Times of Mean Noon at Greenwich and Dehra is the amount the sun's Right Ascension has increased in $5^h\ 12^m\ 13^s$. Suppose therefore we want the Sidereal Time of Local Mean Noon at Dehra Dun on 1st August 1929 We know that Dehra Mean Noon is about 5 hrs earlier than Greenwich and therefore between the Greenwich Mean Noons of 31st July and 1st August, 1929.

		h	m	s
Take Sidereal Time at G.M.N. from N.A. on	31st a	8	34	29.27
....	1st b	8	38	25.83
Difference.			-3	56.56

Now $5^h\ 12^m\ 13^s$ 5.2^h approx.

Multiply $a - b$ by $\frac{5.2}{24}$ Result $c = -51^s.25$

Then $b + c$ (algebraic sum) $= 8^h\ 37^m\ 34^s.58$ = Sidereal Time L.M.N. at Dehra Dun on 1st August 1929.

26. <u>Comparison of clocks and watches</u>. It is required to compare an ordinary mean time watch set in Standard time with a Sidereal clock in the Observatory at Dehra Dun on the same date, viz. 1st August 1929, before starting star observation.

We make two comparisons of the watch and clock thus:-

	h	m	s	h	m	s
Sidereal clock	15	59	27	16	1	25
M.T. watch	7	35	35	7	37	33
	8	23	52	8	23	52

The difference being the same in each case shows that our readings have been correctly taken.

Taking the second observation:-

h	m	s	
16	1	25	
	-	48	Rate correction to Sidereal clock obtained from observatory
16	0	37	Time Sidereal Time of observation
8	37	34.58	S.T. of L.M.N. from para 25
7	23	02.42	Time after noon in intervals of S.T.

	h	m	s		
(Conversion by Table p 562 N.A. 1929,	6	58	51.193	or Conversion by Table 23 Sur Auxiliary Table Part III	7 23 02.42
	0	22	56.232		- 1 8.807
	0	0	1.995		- 3.768
	0	0	0.419		- .006
					- 1 12.581
	7	21	49.84	True L.M.T.	7 21 49.84
	+0	17	47.00	(For Standard Time difference between ℓ 5h. 12m. 13s. Dehra and standard ℓ 5h 30m	
	7	39	36.84	True watch time (standard)	
	7	37	33.00	Watch time observed	
	0	2	3.84	Watch error slow	

27. If again an observation is to be made at Dehra Dun at 20 hrs Standard Time on 1st August 1929 and we want to find the Local Sidereal Time of observation.

	h m s	
	20 0 0	Standard Time
	- 0 17 47	(for difference between ℓ 5h 12m 13s Dehra and Standard ℓ 5h. 30m.
	19 42 13	→ L.M.T.
	-12 0 0	
	or 7 42 13	p.m.
	7 43 28.93	Converted to Sidereal equivalents by Table p 560 N.A. 1929 or Table 22 Sur
	8 37 34.58	S.T. of L.M.N. from para 25
Sum	16 21 03.51	L.S.T. of observation required.

Conversely given Sidereal Time of Observation 16h 21m 3.51s to find the Standard Time of Observation at Dehra Dun on 1st August 1929.

h m		
16 21 3.51	S.T. of Observation	
8 37 34.58	S.T. of L.M.N. from para 25	
7 43 28.93	Difference	
7 42 13	Converted to M.T. equivalents by Table p 562 N.A. or Table 23 Sur	
+12	For 24 hrs system of time	h m s
+17 47	Correction for difference between Dehra and Standard 5h. 30m.	5 12 13
20 00 00	Standard Time of observation required.	

28. Let us now introduce a star

<u>The star α Aquilae is observed E of the meridian at Dehra Dun at 20 hrs Standard Time on 1st August 1929 find its Hour Angle</u>.

As in para 27	16 21 03.51	(Sidereal Time of observation or the (Right Ascension of the Meridian
	19 47 21.46	Right Ascension of star from p 404 N.A. 1929
Difference	3 26 17.95	Hour Angle

<u>Conversely given the Hour Angle 3h 26m 17.94s of the star α Aquilae to find the Standard Time of observation at Dehra Dun on 1st August 1929</u>.

	19 47 21.46	R.A. of star α Aquilae
	3 26 17.95	Hour Angle do
Difference	16 21 03.51	Sidereal time of observation

whence the Standard Time of Observation is found as in para 27.

29. <u>If required, Local Sidereal Time may be converted to Local Apparent Time and vice versa by one of two methods.</u>

<u>1st Method</u>. Apply Equation of Time to L.A.T. and bring it into L.M.T. Turn the L.M.T. into L.S.T., as already explained in paras 24 and 27.

<u>Conversely</u>: Turn the L.S.T. into L.M.T. as in the last 4 lines of the working in para 27. Apply the Equation of Time to the L.M.T. to obtain L.A.T.

<u>2nd Method</u>: L.S.T. = R.A. of Sun + L.A.T. and L.A.T. = L.S.T.-R.A of Sun. The R.A. of the Sun is given every day in the N.A. for Greenwich and we have to interpolate for any other place for the <u>R.A. at Greenwich time corresponding to time of</u> observation.

Hints on Astronomical Observations with the Theodolite.

30. Set up the theodolite complete, tighten the joints, see the legs firmly planted in the ground and have it ready at least 15 minutes before the time when observations are to commence. Focus the micrometers, see the slow motion and footscrews are in the centre of their runs, adjust the eyepiece for parallax and focus telescope on a star. If observing the sun by day, see that the dark glass is affixed to the eyepiece.

If observing by night, erect the sights. The axis reflector and lamp on many theodolites are unsatisfactory. The diaphragm can be more effectively lighted in some cases by means of a lamp pointed towards a small metal reflector, or a piece of stiff white paper can be fastened over the objective with a rubber band, so as to illuminate the cross-hairs without obscuring the view. Level the instrument and see that the top bubble remains readable during the observations.

31. The stars for observation can generally be identified by means of a star chart, but, as the sight vanes on the telescope usually have a slight error, it often requires practice for an observer to pick up the correct star, unless it is a large one. The sight vanes require illuminating by a lamp from behind, while the observer searches for a star. The approximate time and altitude of a star can also be worked out beforehand, if considered necessary, and the star picked up by merely slightly rotating the telescope in azimuth.

32. If no chronographic method is available for noting the time when the sun or star transits the cross-wires, there are several methods by which the time may be estimated and recorded to the nearest tenth of a second.

The booker may merely keep his eye on the chronometer, while the observer says "up" at the time of transit, the booker noting the time to the nearest tenth of a second, or the booker may count seconds and the observer interpolate the time mentally to the nearest tenth of a second. When there is no booker, the observer must necessarily "carry the count" of seconds to or from the reference chronometer, either mentally, or with the aid of a separate stop watch. It is best to start the stop watch as the star crosses the wire, and stop it as soon as possible afterwards, on some particular second of the reference chronometer, reckoning the time of the observation backwards from this point.

33. The booker should warn the observer to be ready to commence about 8 minutes before the time actually down in the programme for each star.

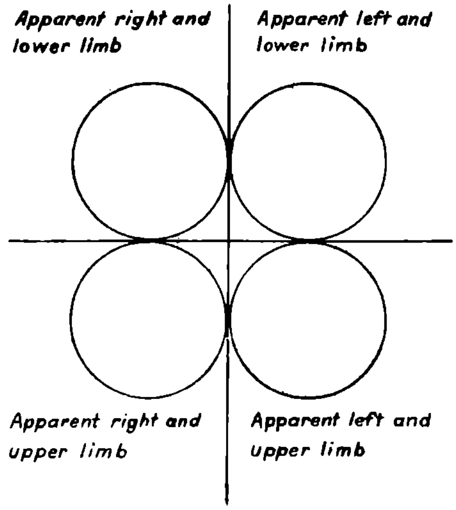
Wait for the booker to say 'right' before giving him any readings. The sequence in which readings should be given (according to the nature of the observations in hand) should be as follows:- first "times", then level readings, object-end eye-end; vertical angles and then horizontal angles. -Vide however footnote p 25.

The booker should examine the readings throughout the observations and warn the observer of discrepancies. The altitude of a star varies but little during a series of observations, and any large discrepancies generally indicate that 2 different stars are being observed.

34. In observing, when possible, always let the heavenly body make contact with the wires with the hands off the theodolite. When contact must necessarily be made by the tangent screw, as in azimuth or latitude observations, do not press on it, so as to disturb the level.

In the case of sun azimuths, where contacts have to be observed in opposite quadrants as shown in the diagram - get the sun into the quadrant required and work with one tangent screw only, so as to keep the vertical wire in contact with the apparent (right or left) limb of the sun, as necessary, and let the other limb (upper or lower), make its own contact. Similarly in observing a star azimuth by the method in which the star is made to pass through the centre of the cross-wires, — work with one tangent screw only and keep the vertical wire on or a little in advance of the star, so that when the star reaches the horizontal wire it will of itself pass through the centre of the cross-wires.

Fig 8.



35. Immediately before and after observing, record the barometer and thermometer readings. The record in the angle book should be complete so that the results can be worked out by an independent person, if necessary.

The information that should be given in the angle book should include, according to the type of observation :-

(a) Nature and purpose of observations
(b) Date and place with approximate latitude and longitude
(c) Number of theodolite & description.
(d) Chronometer used - rate and corrections, if known
(e) Barometer and thermometer readings
(f) Name of object and face and limb observed (for sun only)
(g) E or W, N or S of zenith, if passing the meridian
(h) Referring mark used and its horizontal readings
(i) Observed times
(j) Level readings
(k) Observed altitudes
(l) Horizontal angles of object observed

(m) State of weather and sky
(n) Statement whether observations are good or not.

It is also useful to note the vertical and horizontal collimation error of the theodolite and position of the vertical circle for which the algebraic signs hold good, as affording a check on the angles and enabling single observations to be worked out in doubtful cases in order to locate an error in observing or recording.

Time-keepers. For astronomical observations a good watch or chronometer (preferably sidereal) with a uniform rate is essential. The rate in no case should exceed 12 secs per day. Time observations to ascertain the chronometer error are usually taken at the beginning and end of a programme, so that the error can be applied to intermediate observations such as those for latitude, azimuth, etc. A large rate in a chronometer, even if uniform, is very inconvenient, as it has to be applied throughout the observations. Observations should be taken as quickly as consistent with precision in view of chronometer rate, as well as of the fact that, strictly speaking, star altitudes, etc. do not vary linearly with time except for short periods, as, for instance, is assumed, when a mean altitude, from two face right and two face left observations with a theodolite to a star, is taken for computation purposes as corresponding to a mean time.

Note re theodolite eyepieces . In Sun observations it is best to record the apparent and not the true limbs of the Sun observed, and to state in the angle book whether the eyepiece reversed vertically, horizontally or both. Direct eyepieces reverse both vertically & horizontally, but diagonal eyepieces may do one or the other, which can be easily ascertained by observing the Sun's movement. This sometimes causes mistakes in Sun observations, when only one limb is observed and correction for Sun's semi-diameter is applied.

38. Corrections to observed altitudes. The following corrections have to be applied to the altitudes of heavenly bodies observed by theodolite, before they can be used for computation.

(a) For level error of theodolite, vide para 101 Chapter III Topo Handbook.

(b) For refraction. Refraction makes a heavenly body appear higher than it actually is,

therefore refraction correction is $\frac{\text{subtractive from altitude}}{\text{additive to Z.D.}}$

To obtain refraction correction we should know the barometer and thermometer readings at the time of observation, which should be booked by the observer.

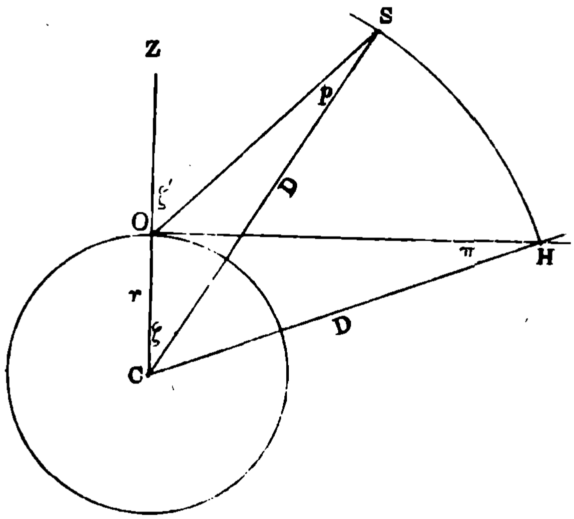
The correction is obtained from Tables 19 & 20 Sur. Auxy Tables Part III 1928.

Refraction is liable to variation and does not always correspond with its tabular values. In order to minimize the effect of using erroneous values, it is advisable to balance an observation to an E star by one to a W star, or one to a N star by one to a S star, so as to eliminate the effect of refraction in the mean of observations. Refraction increases rapidly at low altitudes and its value becomes uncertain, so that observations should never be taken to heavenly bodies below 10° in altitude, and preferably not to those below 15° in altitude.

(c) For semi-diameter. When the sun is the object observed, it is usual to intersect the upper and lower edges or limbs of the sun, and to take the mean of the Z.D's., so derived, as the Z.D. of the sun's centre at the mean of the times when it is observed. This is the method always to be recommended, but it sometimes happens that only one limb is observed and the correction for semi-diameter has to be applied from p II of the N.A. for each month to reduce it to the sun's centre. Mistakes are apt to occur with reversing eyepieces (vide para 37), so it is best to record what you actually see Ω or $\mho$

(d) For parallax. As the sun is not so far away from the earth as a star, a correction for geocentric parallax has to be applied, when the sun is the object observed, as already mentioned in para 1, page 1.

Fig 9



The observed Z.D. is the $\angle$ ZOS instead of the $\angle$ ZCS. The difference, the $\angle$ OSC is the geocentric parallax and is subtractive from Z.D's.

It may be noted that refraction and parallax corrections are of opposite sign.

When the sun is on the observer's horizon, its parallax is at its maximum and is called horizontal parallax ($\angle$ OHC)

In the figure, if π be the horizontal parallax, p the parallax of the Sun at any other Z.D., ζ (ζ' being its observed value), r the earth's radius, and D the distance of the Sun.

We have

$$\sin \pi = \frac{r}{D}$$

$$\frac{\sin p}{\sin \zeta'} = \frac{r}{D} = \sin \pi$$

If we consider (except in the case of the moon) that, as p, π are small, $\frac{\sin p}{\sin \pi} = \frac{p}{\pi}$ approximately.

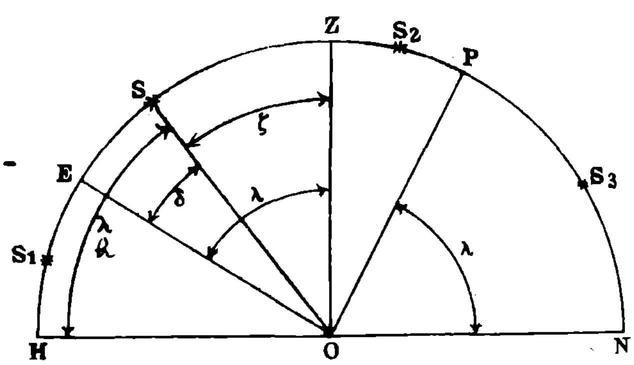
We have $p = \pi \sin \zeta'$

or Parallax = (Horizontal parallax) x (sin Z.D.) ..... (1)
This diminishes with altitude and vanishes at the zenith.
The mean parallax of the sun at various Z.D's is given in Table 21 Sur. sufficiently accurately for all practical purposes.
It can also be obtained from the values given on the first page of the N.A. by means of formula ...... (1)

<u>A note regarding the signs of the quantities δ, λ, ζ etc.</u>

Mistakes sometimes occur in the various problems in field astronomy in applying the correct signs to the above quantities giving rise to resultant errors in the computations.

Fig 10



From fig. 10 it is clear that the latitude of a place λ = decl^n. of the zenith EOZ. It is also equal to the altitude of the pole NOP for ZE = λ and PZ = $90^\circ - \lambda$, & PN = λ. The ZD, ζ is $\pm$ when the heavenly body (S) is $\frac{S}{N}$ of zenith.

Decl^n δ
Latitude of place λ } are $\pm$ when $\frac{N}{S}$ of equator

<u>For observations in N latitude.</u>

If heavenly body (S) is between zenith & equator { ζ is + ve; δ is + ve }
If heavenly body (S_1) is between equator and horizon { ζ is + ve; δ is − ve }

23

If heavenly body (S_2) is between zenith and pole $\begin{cases} \zeta \text{ is -ve} \\ \delta \text{ is +ve} \end{cases}$

If heavenly body (S_3) is below the pole ζ **is -ve** $\delta = 180^\circ$**-true**δ

<u>For observations in S latitude</u> (fig. 11)

If heavenly body be as at S_4, $\lambda = \zeta + \delta$ so that, if $\zeta + \delta$ be given their proper signs, λ, if South, will come out negative.

We thus get the general rule for latitude

$\lambda = \zeta + \delta$, giving

ζ, δ, and λ their proper signs.

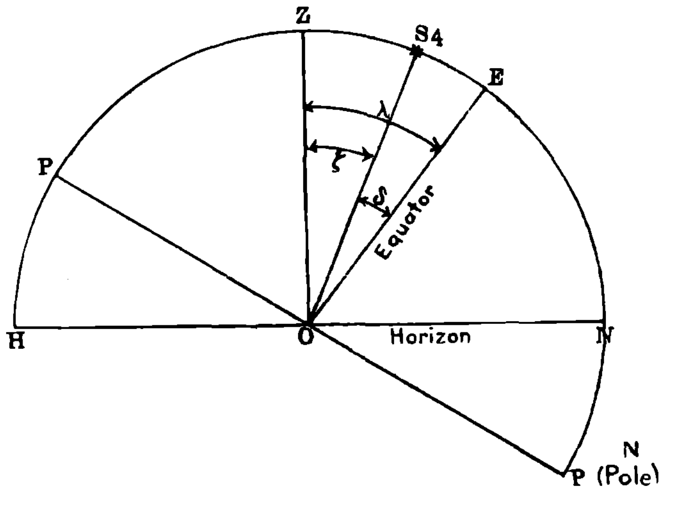
Also colat. $\Upsilon = 90^\circ - \lambda$

$= 90^\circ - \zeta - \delta$ and

$\Delta = 90^\circ - \delta$

so that $\Upsilon = \Delta - \zeta$

Fig 11



Here also ζ has to be given its proper sign according to the rules above so that Υ may be $\Delta \pm \zeta$, as the rule indicates.

In cases of doubt a figure should be drawn in order to decide correct signs to be applied.

---

* δ is here measured from E through the zenith and elevated pole and is equivalent to the angle S_3 OE, that is to say, when a star is taken at its <u>lower</u> transit, the δ is obtained by taking the supplement of the value given in the 'Nautical Almanac'.

Time by observation(Form No.15 Topo.)

39. What is meant by finding the time by observation is finding how much your watch or chronometer is fast or slow. We have already seen that if we know the hour angle of a star, we can find the correct time at which it had that hour angle. Therefore if by any means, we can make an observation which will give the hour angle, and if we note the time at which we made the observation, we can find the true time of the observation, and therefore the amount that the watch or chronometer was fast or slow at that time. We therefore have to carry out an observation that will give us the hour angle.

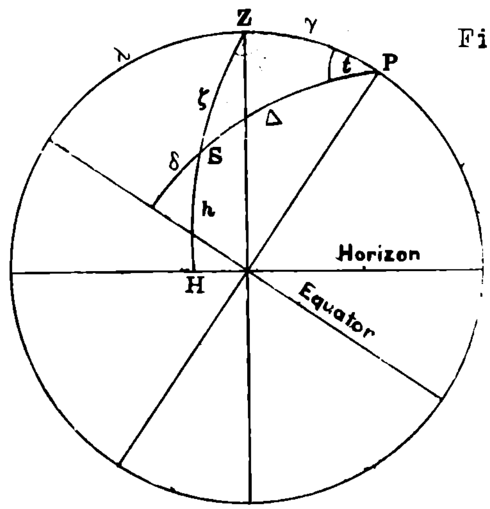


Fig. 12

A glance at the figure will at once show the observation necessary, for, in the △ SPZ, ZP = the Colatitude γ of the place is known; SP = N.P.D. of star is known, = △ SPZ the hour angle t is to be found.

Now with the theodolite it is possible to determine h, the altitude of the star, and ZS = (90° - altitude "h") = ζ

We then have the 3 sides of the △ SPZ and can determine the hour angle 't' by spherical trigonometry.

The procedure is as follows:-

Set up the theodolite - level carefully. Point the theodolite at the star, placing the horizontal wire so that the star will intersect it near the centre of the field. Allow the star to make its own intersection with the horizontal wire and note the time to the nearest 1/10th of a second (vide para 32 p 16).

Read levels* on vertical arc. Read vertical circle.

Change face, point the theodolite at the star & allow the star to make its own intersection with the horizontal wire as before. Note the time to the nearest 1/10th of a second. Read levels* on vertical arc. Read vertical circle. If there are (say) 3 horizontal wires, the time of intersection of each can be noted, provided the same is done on reverse face;and the mean can be taken for the time of intersection of the mean wire. The check of wire intervals is a good indication of the precision with which time is being estimated.

This is a complete observation to one star, but it is usual to take a second observation on the same face,followed by one more with the face of the theodolite reversed to its original position, to balance it and eliminate collimation error. Now in order to obtain refraction correction, we must know the barometer and thermometer readings H & T. These should also be booked at the time of observation.

The method of computation of the results on form 15 Topo is as follows:- First reduce all angular readings to Z.D.'s,and enter the mean observed Z.D.,after correcting for the levels,in form 15 Topo. Enter also the mean value of the chronometer or watch time T, corresponding to the mean Z.D. (on the last line but one of the form).

Now refraction makes a star appear higher than it actually is. Therefore the refraction correction, obtained by means of tables 19 & 20 Sur,is additive to the observed Z.D.

---

* Levels may be read before recording the time,if time is being taken by a stop watch,as there is a chance of the levels moving, while the comparison of the stop watch with reference chronometer is being made.

Having applied these corrections we have the corrected Z.D. of the star,

We also know N.P.D. = ($90 - \delta$, the Declination of the star from N.A.) $= \Delta$; and the Colatitude = $(90 - \lambda) = \Upsilon$,

so that, if t be the hour angle, and we write $2s = (\zeta + \Delta + \Upsilon)$,

we have by ordinary spherical trigonometry:-

$$\tan^2 \frac{t}{2} = \frac{\sin(s-\Delta)\sin(s-\Upsilon)}{\sin s \quad \sin(s-\zeta)},$$

whence t is determined logarithmically in form 15 Topo, and divided by 15 to convert it to time equivalents. Table 34 Sur.Auxy Tables, 1928, Part III, (old 14 Math), is convenient for this conversion.

Let α be the Right Ascension of the star from the N. Almanac.

Local Sidereal Time of observation $\alpha + \frac{t}{15}$, if W of the meridian

$\alpha - \frac{t}{15}$, if E of the meridian

Now look up the Sidereal Time of Greenwich Mean Noon G, (say)

Sidereal interval of time of observation from Greenwich Mean Noon

$\alpha \pm \frac{t}{15} - G$, if $\frac{W}{E}$ of the meridian = m,

and this has to be converted into mean time by applying a retardation for (l-m), where l is the longitude of the place of observation. This method of conversion combines the correction for longitude, converting the Sidereal Time of G.M.N. to L.M.N., with that converting the interval between L.M.N. and the time of observation into mean time equivalents, (vide paras 25, 26 on pages 13, 14).

We thus obtain the true mean time of observation or T_1.

Then $T - T_1$ amount the watch or chronometer is $\frac{\text{fast}}{\text{slow}}$ if $\frac{+}{-}$.

Now in such observations personal errors in timing cannot be eliminated, but other consistent errors in one direction, due to uncertain refraction, slip in the instrument, reading of graduation, etc., may cause Z.D.'s to be observed always too large or too small.

Now if the Z.D. is too great, the hour angle is also too great, so

t the sidereal interval

$\alpha - \frac{t}{15} - G$ is too small, if the star is E. of meridian

$\alpha + \frac{t}{15} - G$ is too large, if the star is W..........

viate this two stars are observed one E and one W and the result is taken.

ean of all the values of $T-T_1$ is taken as the clock error at ean of all the correct times.

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the sun is observed instead of a star, the procedure is some- different.

t we cannot observe sun's centre, so we must observe the edges either:-

Put the horizontal wire along one edge and note the time, and move the wire with the tangent screw and intersect the other and note the time. The mean Z.D. gives the Z.D. of the sun's re at the mean of the times.

If the above is not possible observe only one limb and apply ection for semi-diameter, (vide para 38(c) p 20).

ection for parallax must also be applied, (vide para 38(d) p 21).

r this the computation on form 15 Topo proceeds as before, till btain the hour angle t,

Then t in time =

Equation of time = from N.A. p I

(vide example for method of interpolation in para 24 of these notes)

True mean time of observation = T_1

T = Observed time

$T-T_1$ amount watch or chronometer is $\frac{\text{fast}}{\text{slow}}$ if $\frac{+}{-}$.

1. Now supposing we make a mistake of x in finding ζ, and we want to find in what position of the star this will cause the least mistake y in t;

we have

$$\cos\zeta = \cos\gamma\cos\Delta + \sin\gamma\ \sin\Delta\ \cos t$$
$$\cos(\zeta + x) + \cos\gamma\ \cos\Delta + \sin\gamma\ \sin\Delta\ \cos(t+y)$$

Since x & y are small,

Therefore $x \sin\zeta = y \sin\gamma \sin\Delta \sin t$

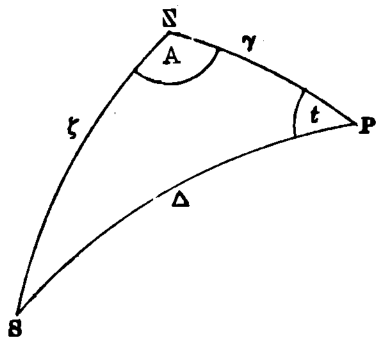
$$y = \frac{x \sin\zeta}{\sin\gamma\ \sin\Delta \sin t}$$

But $\frac{\sin\zeta}{\sin t}\ \left(= \frac{\sin\gamma}{\sin PSZ}\right) = \frac{\sin\Delta}{\sin A}$ where A is Azimuth.

Therefore $\sin\zeta = \frac{\sin\gamma \sin t}{\sin\ PSZ} = \frac{\sin t \sin\Delta}{\sin A}$

$$y = \frac{x}{\sin PSZ \sin\Delta} = \frac{x}{\sin A \sin\gamma}$$

Fig 13



Therefore y will be least when sin A is greatest i.e. when Azimuth A is 90°, or when the star is on the prime vertical. Therefore for this observation stars should be as near the prime vertical as possible, and their altitude not less than 10°, as otherwise uncertainties of refraction come in.

---

2. Occasionally, in some out of the way place, there may be some doubt about the latitude and it may be well to show how this affects the time.

Let t = true hour angle corresponding to true latitude λ

$t + y$ = the hour angle .... latitude $\lambda + x$

Then $\cos\zeta = \cos\Delta \sin\lambda + \sin\Delta \cos\lambda \cos t$

and $\cos\zeta = \cos\Delta\sin(\lambda + x) + \sin\Delta\cos(\lambda + x)\cos(t + y)$,

where x and y are small.

$$= \cos\Delta\sin\lambda + \sin\Delta\cos\lambda\cos t$$
$$+ x\cos\Delta\cos\lambda - x\sin\Delta\sin\lambda\cos t - y\sin\Delta\cos\lambda\sin t$$

By subtracting the above equations

$$\frac{y}{x} = \frac{\cos\Delta\cos\lambda - \sin\Delta\sin\lambda\cos t}{\sin\Delta\cos\lambda\sin t}$$

But $\cos t\sin\lambda = \cos\lambda\cot\Delta - \sin t\cot A$, where A is the ...

" 29 line 6 from top, for cos A at the end of the numerator on the right-hand side read cot A.

x

$\sin\Delta\cos\lambda\sin t$

$$= \frac{\cot A}{\cos\lambda}$$

If then we have an east and a west star, this error will have an opposite sign for them, and the mean error

$= \left[\cot A_e - \cot A_w\right] \frac{x}{\quad}$, in arc;

" ... line ... from top, read the equation as
$\cos(\zeta + x) = \cos\gamma\cos\Delta + \sin\gamma\sin\Delta\cos$

known, it is advantageous to select 2 stars whose azimuths, E and W, are nearly the same.

Approximate method of obtaining Time and Latitude by observation.

If neither time, latitude nor azimuth are known, latitude could be accurately obtained, if we had a theodolite in perfect adjustment, so that there was no collimation error in either the horizontal or vertical wires, and we could place the theodolite with its vertical wire in the meridian.

All that would then be necessary, would be to observe the Z.D. ζ_0 of a star at that instant when it crossed the vertical wire. This Z.D ζ_0 corrected for refraction, and added to or subtracted from Δ, the N.P.D., would at once give the Colatitude γ, (vide note p 22, 23), and $\cos \zeta = \cos\Delta \sin(\lambda + x) + \sin\Delta \cos(\lambda + x) \cos(t + y)$,

where x and y are small.

$$= \cos\Delta \sin\lambda + \sin\Delta \cos\lambda \cos t$$
$$+ x \cos\Delta \cos\lambda - x \sin\Delta \sin\lambda \cos t - y \sin\Delta \cos\lambda \sin t$$

By subtracting the above equations

$$\frac{y}{x} = \frac{\cos\Delta \cos\lambda - \sin\Delta \sin\lambda \cos t}{\sin\Delta \cos\lambda \sin t}$$

But $\cos t \sin\lambda = \cos\lambda \cot\Delta - \sin t \cot A$, where A is the azimuth

$$\frac{y}{x} = \frac{(\cos\Delta \cos\lambda - \cos\Delta \cos\lambda) + \sin\Delta \sin t \cos A}{\sin\Delta \cos\lambda \sin t}$$

$$= \frac{\cot A}{\cos\lambda}$$

If then we have an east and a west star, this error will have an opposite sign for them, and the mean error

$$= \left[\cot A_e - \cot A_W\right] \frac{x}{2 \cos\lambda}, \text{ in arc};$$

so that, if there is reason to fear the latitude is not well known, it is advantageous to select 2 stars whose azimuths, E and W, are nearly the same.

Approximate method of obtaining Time and Latitude by observation.

If neither time, latitude nor azimuth are known, latitude could be accurately obtained, if we had a theodolite in perfect adjustment, so that there was no collimation error in either the horizontal or vertical wires, and we could place the theodolite with its vertical wire in the meridian.

All that would then be necessary, would be to observe the Z.D. ζ_0 of a star at that instant when it crossed the vertical wire. This Z.D ζ_0 corrected for refraction, and added to or subtracted from Δ, the N.P.D., would at once give the Colatitude γ, (vide note p 22, 23), or, if we had a perfectly adjusted theodolite and knew the time accurately, all that would be necessary would be to keep the star on the horizontal wire and stop at the correct time of meridian transit. Though we never have a theodolite in perfect adjustment as regards collimation, we can adjust the instrument for collimation error as far as possible, and find the amount and sign of the residual correction, positive or negative, to an angle observed on a particular face. We can then obtain an approximate latitude as well as time as follows:-

Set up the theodolite and intersect Polaris. Clamping the horizontal circle, swing the telescope on its transit axis and select a star, recognisable in the star chart, of south aspect and of convenient zenith distance which is a little east of the vertical place of the telescope. Wait till this is observable in the telescope, and then take its Z.D. If the Z.D. is diminishing, it is clear that the star has not yet reached the meridian, it can then be followed up until the star no longer appears to rise, and the Z.D. remains stationary, when the star is in the meridian, and the watch time of this, t, and the Z.D. at the same moment, ζ_0, corrected for level, collimation and refraction may be entered in the equations

$$\Delta - \zeta = 90^\circ - \lambda$$
$$\delta + \zeta = \lambda$$

(ζ and δ with proper signs. vide note p 22, 23)

and R.A. — t watch error, slow if + fast if —

in which Δ is north polar distance of the star, δ is its declination, R.A. is its right ascension and λ is the latitude of the place.

If, however, the Z.D. of the star, when first observed, is found to be increasing, it has clearly passed the meridian, and a slightly more easterly star should be immediately selected, and observed with the telescope swung a degree or two to the east.

For subsequent pairing of the star it will often be convenient if the Z.D. is about the same as that of Polaris, if the latitude is not less than 15°, when uncertainties of refraction come in.

The above method should give a value of latitude correct to perhaps 30", and a rough value of time correct to less than a minute, and has the advantage that no logarithmic computation is necessary and the results are available at once. As however we never have an instrument in perfect adjustment, and we cannot take a face right & face left observation at the same time, we have to resort to another method in order to obtain latitude more precisely, viz:-

<u>The method of Circum-meridian Observations for Latitude, Form 13 Topo.</u>

In this observation we observe the zenith distance (Z.D.) of a star, when it is near the meridian and apply a correction to reduce it to the meridian.

The true time that the star crosses the meridian is found by subtracting the Sidereal Time of Mean Noon from the R.A. of the star and, if necessary, reducing to mean time. If we apply to this the clock error e_0 at this time, we get the clock time of the star's transit over the meridian. Call this T_0.

Also if T_1 be the clock time at which we observe the star, T_1 = True time of observation, the clock error corresponding being e_1. Now the interval $T_1 - T_0$ is small, so that unless the clock rate is very large, $e_0 = e_1$

$$T_0 - T_1 = \text{True time of transit} - \text{true time of observation}$$

$$= t, \text{ the star's hour angle.}$$

Now $\cos \zeta = \cos \gamma \cos \Delta + \sin \gamma \sin \Delta \cos t.$

and $\cos t = 1 - 2\sin^2 \frac{t}{2}$

Therefore $\cos \zeta = \cos(\gamma - \Delta) - \sin\gamma \sin\Delta \, 2\sin^2 \frac{t}{2}$

Now $\gamma - \Delta$ = meridian Z.D.

$= \zeta \pm x$ where x is small.

$\cos(\gamma - \Delta) = \cos(\zeta \mp x) = \cos\zeta + x \sin 1'' \sin\zeta$

Therefore $x = \frac{\sin\gamma \sin\Delta}{\sin\zeta} \; \frac{2\sin^2 \frac{t}{2}}{\sin 1''}$ .... (a)

The values of $m = \frac{2\sin^2 \frac{t}{2}}{\sin 1''}$ are given in Table 24 Sur. Auxy Table Part III 1928.

The above formula (a) is that used in form 13 Topo to get A (m) or C_1, the first correction, in lines 37-43 of the form.

A secondary correction is given in Tables 25, 26 and 27 Sur. Auxy. Tables, Part III 1928, which may be omitted if the intervals t fr transit are kept sufficiently small, (vide explanations on form 13 Topo and on p 35 Auxy. Tables Part III 1928).

In practice you observe the Z.D. on F.R., note the time to the nearest tenth of a second
.... F.L. ...."

The mean Z.D. corrected for refraction and level,
= Z.D. corresponding to mean of times t_1.

Two more observations give Z.D. ζ_1 corresponding to t_2 and so on.

Five or six such sets should be taken and the nearer they are to the time of transit the better.

None of the t's should be greater than 20 mins, and, by taking some the observations on one side and some on the other side of the meridian, it is possible to get all the t's less than 10 mins.

Beginners ought to compute out each set separately, and the accordance of the results will furnish a check. There should not be more than between the greatest and least of them. Afterwards, when the observer is quite familiar with the method it is quite sufficient to take mean of the ζ's as corresponding to the mean of the m's, not the

Now $\cos \zeta = \cos \Upsilon \cos \Delta + \sin \Upsilon \sin \Delta \cos t$.

and $\cos t = 1 - 2 \sin^2 \frac{t}{2}$

Therefore $\cos \zeta = \cos (\Upsilon - \Delta) - \sin \Upsilon \sin \Delta \; 2 \sin^2 \frac{t}{2}$

Now $\Upsilon - \Delta$ = meridian Z.D.

= $\zeta (+) x$ where x is small

" 32 line 5 from top, for $\xi + x$ read $\xi - x$, and after "x is small" add [Here it is convenient to ignore the rule of signs given on page 22.]

-- $\frac{\ldots}{\sin \zeta}$ $\frac{\ldots \sin \frac{t}{2}}{\sin 1''}$ .... (a)

The values of $m = \frac{2 \sin^2 \frac{t}{2}}{\sin 1''}$ are given in Table 24 Sur. Auxy Tables Part III 1928.

The above formula (a) is that used in form 13 Topo to get A (m) or C_1, the first correction, in lines 37-43 of the form.

" 32 line 8 from top, for $\cos(\xi + x)$ read $\cos(\xi - x)$

... Sur. Auxy. Tables, Part III 1928, which may be omitted if the intervals t from transit are kept sufficiently small, (vide explanations on form 13 Topo and on p 35 Auxy. Tables Part III 1928).

In practice you observe the Z.D. on F.R., note the time to the nearest tenth of a second

.... F.L. ....

The mean Z.D. corrected for refraction and level,
= Z.D. corresponding to mean of times t_1.

Two more observations give Z.D. ζ_1 corresponding to t_2 and so on.

Five or six such sets should be taken and the nearer they are to the time of transit the better.

None of the t's should be greater than 20 mins, and, by taking some of the observations on one side and some on the other side of the meridian, it is possible to get all the t's less than 10 mins.

Beginners ought to compute out each set separately, and the accordance of the results will furnish a check. There should not be more than between the greatest and least of them. Afterwards, when the observer is quite familiar with the method it is quite sufficient to take the mean of the ζ's as corresponding to the mean of the m's, not the mean of the t's, and compute out one single deduction.

If a mean time chronometer is used for observation, t's should be converted into Sidereal Time intervals by 22 Sur, before m's are taken out from the table.

In form 13 Topo an approximate value of meridian Z.D. is used. This is obtained from the mean of the single pair of observations, occurring nearest the time of transit. Then $\gamma_a = \Delta - \S$.

The other steps in Form 13 Topo should be clear from the explanations on the form itself and preceding notes.

If the sun is the object observed, the altitude must be measured to the upper and lower limbs alternately, and mean values taken as referring to the centre. Parallax corrections must also be applied to sun observations.

When a star crosses the meridian on the $\frac{\text{same}}{\text{opposite}}$ side of the pole as the zenith, at the time of its transit it is said to be at $\frac{\text{upper}}{\text{lower}}$ culmination.

Consider a star at upper culmination and North of the zenith, then if we make a mistake in reading the Z.D.,

$$\text{Colatitude} = \S + x + \Delta = \text{True value } \gamma + x.$$

If however the star is at upper culmination and S of the zenith and the same mistake is made,

$$\text{Colatitude} = \Delta - \S - x$$

$$= \text{True value } \gamma - x$$

Therefore the mean is free from error.

We thus see that 2 stars should be used, one S and the other N of zenith, and the latter either at lower or upper culmination, the star's position being subject to the restriction that the star must be high enough to avoid errors of refraction. Luckily we have a star which answers admirably down as far as latitude 20° and that is Polaris.

Below 20° stars at upper culmination, one N and the other S of the zenith, should be used.

General remarks regarding observations to Polaris for latitude or azimuth.

Time cannot be determined with precision by Polaris: and conversely precise time is not an essential for the reduction of observations to Polaris. Thus an error of 3 seconds of time will seldom cause an error of 1" in azimuth, and less in latitude. From this it can be seen how nearly the time should be known for any particular accuracy of deduction. Approximate values of the time or latitude are required in the observations for latitude or azimuth.

Observation for latitude from Polaris out of the meridian, the time being known, Form 14. Topo.

Having obtained time by a time observation either precisely as in paras 39-40, or roughly, as in para 43, intersect Polaris on both faces with the horizontal wire of the theodolite, noting the time. The mean of 4 intersections will give a good result.

The computation of the observations to Polaris is simplified by the fact that the N.P.D. of Polaris is small itself, viz:- about $1\frac{1}{2}^\circ$

We have:- $\cos\zeta = \cos\gamma\cos\Delta + \sin\gamma\sin\Delta\cos t$

Let latitude = altitude − x, where x is small

$$= h - x$$

$$= 90^\circ - \zeta - x$$

or colatitude $\gamma = \zeta + x$

$$\cos\zeta = \left[\cos\zeta\left(1-\frac{x^2}{2}\right) - \sin\zeta\left(x-\frac{x^3}{6}\right)\right]\left(1-\frac{\Delta^2}{2}\right)$$

$$+\left[\sin\zeta\left(1-\frac{x^2}{2}\right) + \cos\zeta\left(x-\frac{x^3}{6}\right)\right]\left(\Delta-\frac{\Delta^3}{6}\right)\cos t$$

$$= \cos\zeta - x\sin\zeta - \frac{\cos\zeta}{2}(x^2+\Delta^2) + \sin\zeta\left(\frac{x^3}{6}+\frac{x\Delta^2}{2}\right)$$

$$+\Delta\sin\zeta\cos t + x\Delta\cos\zeta\cos t - \sin\zeta\cos t\left(\frac{x^2\Delta}{2}+\frac{\Delta^3}{6}\right)$$

$$x = \Delta\cos t - \frac{1}{2}\cot\zeta\,(x^2+\Delta^2-2x\Delta\cos t)$$

$$+\frac{1}{6}(x^3+3x\Delta^2-3x^2\Delta\cos t-\Delta^3\cos t)$$

1st approx. $x = \Delta\cos t$

2nd approx. $x = \Delta\cos t - \frac{1}{2}\Delta^2\sin^2 t\cot\zeta$

is Polaris.

Below 20° stars at upper culmination, one N and the other S of the zenith, should be used.

General remarks regarding observations to Polaris for latitude or azimuth.

Time cannot be determined with precision by Polaris: and conversely precise time is not an essential for the reduction of observations to Polaris. Thus an error of 3 seconds of time will seldom cause an error of 1" in azimuth, and less in latitude. From this it can be seen how nearly the time should be known for any particular accuracy of deduction. Approximate values of the time or latitude are required in the observations for latitude or azimuth.

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The computation of the observations to Polaris is simplified by the fact that the N.P.D. of Polaris is small itself, viz:- about $1\frac{1}{2}^\circ$.

We have:- $\cos\xi = \cos\gamma\cos\Delta + \sin\gamma\sin\Delta\cos t$

Let latitude $= \text{altitude} - x$, where x is small

$= h - x$

$= 90^\circ - \xi$ [illegible] x

" 34 line 3 from bottom, the bracket] to be placed as follows:-

$$\cos\xi = \left[\cos\xi\left(1-\frac{x^2}{2}\right) - \sin\xi\left(x-\frac{x^3}{6}\right)\right]\left(1-\frac{\Delta^2}{2}\right)$$

$$+\left[\sin\xi\left(1-\frac{x^2}{2}\right) + \cos\xi\left(x-\frac{x^3}{6}\right)\right]\left(\Delta-\frac{\Delta^3}{6}\right)\cos t$$

$$= \cos\xi - x\sin\xi - \frac{\cos\xi}{2}(x^2+\Delta^2) + \sin\xi\left(\frac{x^3}{6}+\frac{x\Delta^2}{2}\right)$$

$$+\Delta\sin\xi\cos t + x\Delta\cos\xi\cos t - \sin\xi\cos t\left(\frac{x^2\Delta}{2}+\frac{\Delta^3}{6}\right)$$

$$x = \Delta\cos t - \frac{1}{2}\cot\xi\,(x^2+\Delta^2-2x\Delta\cos t)$$

$$+\frac{1}{6}(x^3+3x\Delta^2-3x^2\Delta\cos t-\Delta^3\cos t)$$

1st approx. $x = \Delta\cos t$

2nd approx. $x = \Delta\cos t - \frac{1}{2}\Delta^2\sin^2 t\cot\xi$

Put this in the 2nd term and 1st approx. in third.

3rd approximation

$$x = \Delta\cos t - \tfrac{1}{2}\Delta^2 \sin^2 t \cot \S + \tfrac{1}{3}\Delta^3 \cos t \sin^2 t$$

Now the greatest value $\cos t \sin^2 t$ can have is when $3\cos^2 t = 1$ and as Δ is less than the circular measure of $1^\circ.30'$ the last term is less than $\frac{1}{2}''$ — of arc, and may be neglected unless great accuracy is required.

Therefore latitude $= (90^\circ - \S) - \Delta\cos t + \frac{1}{2}\Delta^2 \sin^2 t \cot \S$

These are all in circular measure, therefore in arc

Latitude $= (90^\circ - \S) - \Delta''\cos t + \frac{1}{2}\Delta''^2 \sin^2 t \cot \S \sin 1''$

The computation of α or $\Delta''\cos t$ is effected in lines 25 to 27 of form 14 Topo.

The computation of β (line 29 of form 14 Topo) or $\frac{1}{2}\Delta''^2 \sin^2 t \cot \S \times \sin 1''$ is effected by means of Table 28 Sur. Auxy. Tables. Part III 1928, instead of by Tables, which now only appear in the abridged Nautical Almanac.

Table 28 Sur. gives values of $\beta_0 = \frac{1}{2}(3960)^2 \sin^2 t \cot \S \sin 1''$ for N.P.D. $1^\circ.6'$, 3960 being the number of seconds in the standard N.P.D $1^\circ.6'$ and γ, the correction for difference of 1 minute to N.P.D. from $1^\circ 6'$.

$\beta = \beta_0 + f(\text{correction } \gamma)$, where $f = (\text{N.P.D.} - 1^\circ 6')$ in minutes, (vide also explanation of Table 28 Sur on p XIV Auxy. Tables part III 1928)

The remainder of the computation on Form 14 Topo follows principles already explained and should be easily intelligible.

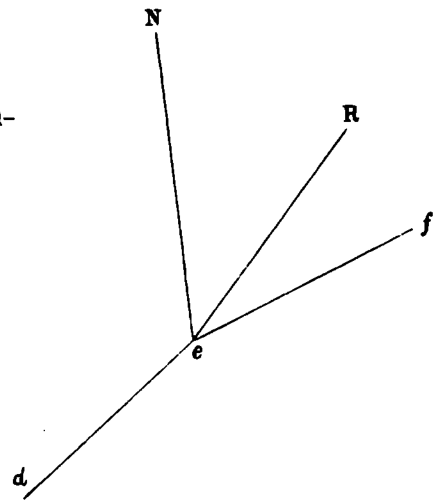
Azimuth by observation.

When we talk of finding our azimuth we mean finding the angle which one side of our triangulation or one ray of our traverse makes with the meridian through one of its extremities. In traverse work where angular errors tend to accumulate in the resultant bearings, an azimuth observation is necessary every 5 to 10 miles, so as to ascertain the accumulated error in bearing and correct the observed angles and bearings. As the azimuth observation unless taken to the sun has to be generally made after dark and as it is not always convenient to set up a lamp at the other extremity of the ray of the traverse or triangulation itself, it is very often necessary to put up a lamp as a referring mark (R.M.) or object in any direction at a convenient distance not less than 400 yards and preferably over ½ a mile from the station of observation, and determine the angle which the ray joining the station of observation to the R.M. makes with the meridian at the station.

The R.M. usually consists of a bull's eye lamp centred on a peg (or pole) driven into the ground and so arranged that the light is directed towards the observer. If the lens of the bull's eye is large it should be covered with a piece of brown paper with a small hole cut in its centre, so as to leave a small point of light only to observe to with the theodolite. The angle between the peg and the ray of the traverse or triangulation whose azimuth is required, can be observed either in the evening before the observation for azimuth or else next morning.

In figure (14), e is the station of observation; ef the next forward ray of our traverse (de, ef) or ray of our triangulation, the azimuth of which is to be determined; and R is the referring mark. By astronomical observation we find the angle N'eR between true north and the R.M., and we also observe the angle Ref between the R.M. and the next forward station. We thus get the angle N'ef the true bearing or azimuth of the forward ray ef, which can be compared with that obtained by triangulation or traverse (after eliminating the correction for convergency of the meridian in the latter case) and the error distributed backwards in the rays of the triangulation or traverse (vide also Sph. Trig. notes p. 19)

Fig. 14



Determination of Azimuth from Polaris. time and latitude being known. Form 25 Topo

Level the theodolite carefully, as the error of dislevelment affects the azimuth. This error increases as the tan(altitude) of the star to which observations for azimuth are taken.

In this case Polaris is the star to which observations are to be taken. The procedure is as follows:-

Intersect the R.M. with the vertical wire, read the horizontal angles. Intersect Polaris with the vertical wire, noting the time, read the horizontal angles. Complete the round of angles by reintersecting the R.M. and again observing the horizontal angles. Change face and repeat the same observations, swinging the theodolite in the opposite direction to that first adopted.

The computation depends on the following approximation.

In the figure S is the star, P the pole, Z the zenith, SN the portion of a great circle through S perpr to PZ.

We have $\sin SN = \sin\Delta \sin t$

$\tan PN = \tan\Delta \cos t$

$\tan A = \tan SN \operatorname{cosec} ZN$

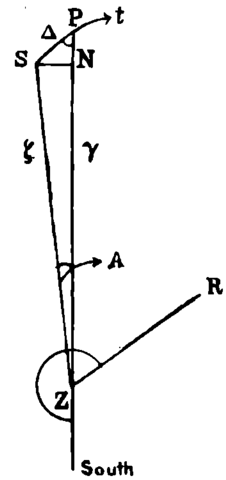
$= \tan SN \operatorname{cosec} (\gamma - PN)$

As Δ and A are small in the case of Polaris, $A = SN \operatorname{cosec} (\gamma - \Delta \cos t)$

$= \Delta \sin t \sec (90^\circ - \gamma + \Delta \cos t)$

$= \Delta \sin t \sec (\lambda + \Delta \cos t)$

Fig. 15



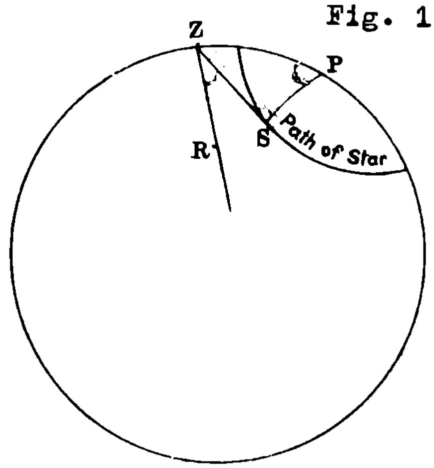
In form 25 Topo we compute first $a = \Delta \cos t$ & then A the azimuth from the formula $\Delta \sin t \sec (\lambda + a)$.

A little care must now be taken to verify whether Polaris was E or W of the meridian at the time of observation, which may be done by comparing the sidereal time of observation against the R.A. The azimuth from south $(180 \pm A)$ is entered in the 28th line of form 25 Topo, the plus sign if east and the minus if west of the meridian and the angle between R.M. & the star S (Polaris) being applied with the correct sign (which is best ascertained from a rough diagram made at the time of observation in the angle book and showing the relative positions of the R.M., Polaris and true north as observed) the azimuth of the referring mark from south is derivrd. In the figure, for instance, Z, the zenith, representing the observer's position, & R the referring mark; $180 - A + \angle SZR$ is the azimuth of R.M. from south in this case.

Azimuth by a circumpolar star at elongation.

If we know the latitude and time, azimuth can be determined by observing the horizontal angle RZS between the referring mark R, and a star S, at Z, the zenith, which represents the observer's position. The observation must be taken on both faces of the theodolite and the times noted at which we intersect the star. From the mean of the times and the star's R.A. we can find the hour angle; and then, from the astronomical triangle SPZ, the azimuth angle SZP can be calculated; and hence the angle RZP, the azimuth of the referring mark R. The question is which is the best star to observe and at what time it should be observed.

Fig. 16



If a star transits between the pole and the zenith, we can draw a tangent from Z to its path so that the angle ZSP is a right angle. When the star is at this point S, it is said to be at eastern or western elongation, according to which side of the meridian it is. When at S, the star is moving directly towards Z, and for some time before and after it reaches S, it is moving very slowly in azimuth with respect to Z. If therefore we can observe the star when it is at S, a considerable error in time will cause a comparatively small error in azimuth. Also the nearer the pole, the slower the star moves, so that the ideal object is a close circumpolar star at elongation. If we are going to observe such a star, we must know when it is at elongation. The sidereal time of the star's transit R.A. - S.T. of L.M.N., and if P be the angle SPZ, we must subtract from this the $\angle$ P, reduced to hours for eastern elongation and add it for western. $\angle P$ is derived from the equation $\cos P = \tan\Delta \tan\lambda$

repeat the same observations, swinging the theodolite in the opposite direction to that first adopted.

The computation depends on the following approximation.

In the figure S is the star, P the pole, Z the zenith, SN the portion of a great circle through S perpr to PZ.

We have $\sin SN = \sin\Delta \sin t$

$\tan PN = \tan\Delta \cos t$

$\tan A = \tan SN \operatorname{cosec} ZN$

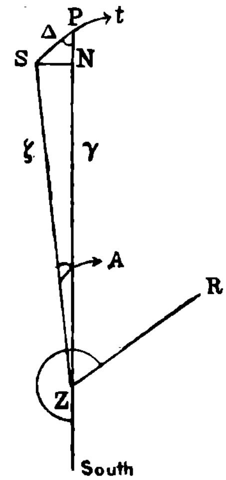
$= \tan SN \operatorname{cosec} (\gamma - PN)$

As Δ and A are small in the case of Polaris, $A = SN \operatorname{cosec} (\gamma - \Delta \cos t)$

$= \Delta \sin t \sec (90^\circ - \gamma + \Delta \cos t)$

$= \Delta \sin t \sec (\lambda + \Delta \cos t)$

Fig. 15



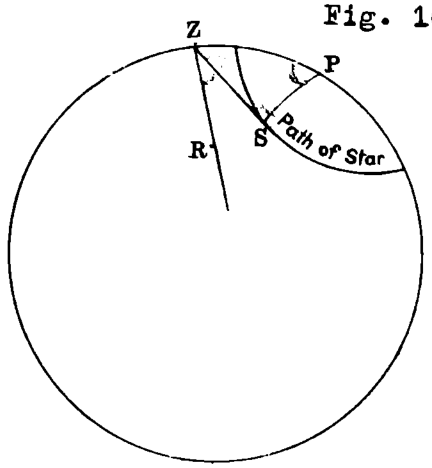
In form 25 Topo we compute first $a = \Delta \cos t$ & then A the azimuth from the formula $\Delta \sin t \sec (\lambda + a)$.

A little care must now be taken to verify whether Polaris was E or W of the meridian at the time of observation, which may be done by comparing the sidereal time of observation against the R.A. The azimuth from south ($180 \pm A$) is entered in the 28th line of form 25 Topo, the plus sign if east and the minus if west of the meridian and the angle between R.M. & the star S (Polaris) being applied with the correct sign (which is best ascertained from a rough diagram made at the time of observation in the angle book and showing the relative positions of the R.M., Polaris and true north as observed) the azimuth of the referring mark from south is derivrd. In the figure, for instance, Z, the zenith, representing the observer's position, & R the referring mark; $180 - A + \angle SZR$ is the azimuth of R.M. from south in this case.

Azimuth by a circumpolar star at elongation.

If we know the latitude and time, azimuth can be determined by observing the horizontal angle RZS between the referring mark R, and a star S, at Z, the zenith, which represents the observer's position. The observation must be taken on both faces of the theodolite and the times noted at which we intersect the star. From the mean of the times and the star's R.A. we can find the hour angle; and then, from the astronomical triangle SPZ, the azimuth angle SZP can be calculated; and hence the angle RZP, the azimuth of the referring mark R. The question is which is the best star to observe and at what time it should be observed.

Fig. 16



If a star transits between the pole and the zenith, we can draw a tangent from Z to its path so that the angle ZSP is a right angle. When the star is at this point S, it is said to be at eastern or western elongation, according to which side of the meridian it is. When at S, the star is moving directly towards Z, and for some time before and after it reaches S, it is moving very slowly in azimuth with respect to Z. If therefore we can observe the star when it is at S, a considerable error in time will cause a comparatively small error in azimuth. Also the nearer the pole, the slower the star moves, so that the ideal object is a close circumpolar star at elongation. If we are going to observe such a star, we must know when it is at elongation. The sidereal time of the star's transit R.A. - S.T. of L.M.N., and if P be the angle SPZ, we must subtract from this the $\angle P$, reduced to hours for eastern elongation and add it for western. $\angle P$ is derived from the equation $\cos P = \tan\Delta \tan\lambda$

Also to find the position of the star(often small) we can determine its altitude at time of elongation which is derived from the equation $\sin h = \sec\Delta \sin\lambda$ (which has to be corrected for refraction)to give the altitude to be set on the theodolite

The azimuth A or ∠SZP is derived from equation $\sin A = \sin\Delta \sec\lambda$
For ordinary purposes it is sufficiently accurate to take an observation or a pair of observations on each face within 5 minutes of the computed time of elongation on either side. The azimuth A is then easily computed from the above formula. To obtain the azimuth of the referring mark, the angle A is added to or subtracted from the angle between the referring and star, the sign $\pm$ being best determined from a diagram drawn in the angle book , showing the relative position of R,S,Z(observer's position) and P at the time of observation. Unfortunately the close circumpolar stars except Polaris are small stars, so that generally a theodolite with a powerful telescope is required for the observation. Also for the highest precision as in Geodetic azimuths, as it is impossible to observe both faces at the exact moment of elongation , a modification of the method has to be made so as to introduce a correction of each observation to the time of elongation. (vide Old Handbook of Proflm Instructions Trigl Branch Ch II 1922).

<u>Azimuth from star observations(East & West) star),time and latitude being known. Form 12 Topo</u>. The"star at elongation"method of the last para is not always possible, as the circumpolar stars except Polaris are small, so that wo are usually compelled to fall back on some other method. The method which at once

suggests itself is to take a star further away from the pole and to take it as near elongation as you can get it.

In this case we have $\cos\gamma \cos t = \sin\gamma \cot\Delta - \sin t \cot A$

Put $\tan\varphi = \tan\Delta \cos t$ and we get

$$\cot A = \cot t \left(\frac{\sin\gamma \cos\varphi}{\sin\gamma\varphi} - \cos\gamma \right)$$

$$= \frac{\sin(\gamma - \varphi) \cot t}{\sin\gamma\ \varphi}$$, a form suitable for logs.

This formula is that given in most text books, but, as it fails when hour angle $t = 90^\circ$, another formula is used in form 12 Topo.

viz:- $$\tan \frac{A-B}{2} = \frac{\sin\frac{\Delta-\gamma}{2} \cot\frac{t}{2}}{\sin\frac{\Delta+\gamma}{2}}$$

$$\tan \frac{A+B}{2} = \frac{\cos\frac{\Delta-\gamma}{2} \cot\frac{t}{2}}{\cos\frac{\Delta+\gamma}{2}}$$

, whence A, the Azimuth is obtained.

Now supposing for example that the R.M. is west of the meridian, the angle R.M. - meridian $= \angle$(R.M.-star) - A, if star is E

But if we make a mistake of x in intersecting the star, we have

$\angle$ R.M.-meridian $= \angle$R.M.-(star-x)- A sif star is east

$\angle$ R.M.-meridian $= A - \angle$[R.M.- (star -x)], if we intersect a second star W of meridian and make the same mistake in intersection.

In the mean x will cancel out and we get the azimuth free from errors of observation. The procedure will then be as follows:- Select two stars E & W as near the pole as possible, and as near elongation as possible. Level the theodolite carefully, as dislevelment increases as the tan (altitude of star). Then (supposing the R.M. is between the two stars) intersect the R.M. first, and read the horizontal angles, then intersect (say) the west star noting the time of intersection & read the horizontal angles, then the east star noting the time of intersection

& read the horizontal angles, then close on the referring mark and read the horizontal angles. Reverse face and repeat the observations on the referring mark, then on the east star, then on the west star and close on the referring mark. It is best in observing these two rounds of angles to work face left, swing right and face right, swing left, respectively. Mean angles of both faces are taken for computing.

The formula on which the computation depends has already been given and the working on form 12 Topo should be clear therefrom.

In traverse work, where great accuracy is not required and where it is not considered advisable to entrust subsurveyors with valuable watches, a method is used where time is not required.

The theodolite is carefully levelled, as before, and the referring mark intersected & horizontal angles read. Then the west star is intersected (say), and, by means of the vertical and horizontal tangent screws, the cross wires are so placed that the star passes through their intersection. The angles on both vertical and horizontal circles are then read, the level on the vertical circle having first been recorded. Then a similar observation is carried out on the east star, the level, vertical & horizontal angles being read and the round is closed on the referring mark and the horizontal angles read. Face is then reversed and observations are carried out with the opposite swing in the reverse order. The means of the readings F.R. and F.L. are used in the computations. We now know the three sides of the astronomical triangle SPZ viz:- Δ, γ and ξ, and A the azimuth is computed on form 11 Topo-by the formula:-

$$\tan\frac{A}{2} = \sqrt{\frac{\sin(s-\gamma)\ \sin(s-\xi)}{\sin s\ \sin(s-\Delta)}}, \quad \text{where } 2s = \Delta + \gamma + \xi$$

The computation should be easily intelligible from the form.

Computation of Azimuth from Sun - Horizontal and Vertical Angles observed simultaneously Forms 11 Topo and 4 Trav.

In a topographical party traversers usually prefer to take sun instead of star observations for azimuth, as this obviates their having to sit up late at night after their day's work. The sun gives sufficiently good results for their purpose, as they should not be in error by more than 30" with a theodolite reading to 30", and an error of 1 minute is permissible in traverse work for 1-inch survey. The variations in procedure in observing both limbs of the sun already referred to on page 18 must be adopted, or, if only one limb of the sun is observed, a correction for the sun's semi-diameter must be applied. For sun observations a dark glass is fitted over the eyepiece of the theodolite. Azimuth should be observed to the sun E, early in the morning, or to the sun W, late in the afternoon; and not within 3 hours of the time it transits the meridian, as, during the middle of the day, it is moving too rapidly in azimuth. The sun's altitude at the time of observation should, if possible, be between 20° and 40°. The 2 forms, on which sun azimuths (from Horizontal and Vertical angles observed simultaneously) can be computed, are forms 11 Topo. and 4 Trav., the latter being simplified for ordinary traverse work.

If the zero of the theodolite be set to magnetic north before the R.M. is intersected, the needle bearing of the latter can be entered at the head of Form 4 Trav., and the difference between this and the working azimuth from north, obtained by computation in the last line of the form, gives the variation of the needle.

The order of observations in the field usually is as follows:-

Referring object or mark (R.O. or R.M.) - $Horiz^1$ arc read.

Sun (Apparent right & upper limbs) - Level, $Vert^1$ & $Horiz^1$ arcs read.

Face & swing changed.

Sun (Apparent left & lower limbs) - Level, $Vert^1$ & $Horiz^1$ arcs read.

R.O. or R.M. - $Horiz^1$ arc read.

The quadrants, in which the sun is observed above, are for afternoon observations. For morning observations, the sun would be observed in the other quadrants, and sun (Apparent right and lower limbs) and (Apparent left and upper limbs) would be the observations to be recorded.

The observations may be repeated on a second zero.

An alternative method is to repeat the readings to each limb of the sun before the second intersection of the R.M., instead of taking a second round on another zero.

The formula, on which form 11 Topo. is based, has already been explained, and that used in form 4 Trav. is the same. The only differences in form 4 Trav. are simplifications, as the form does not aim at such a high standard of accuracy as form 11 Topo. Thus in form 4 Trav. refraction is taken from table 42 Sur. Part III Aux Tables 1928, for a fixed barometer pressure of 29 inches and temperature 75°F; whereas in form 11 Topo it is computed from tables 19 and 20 Sur. for the actual barometric pressure and temperature recorded at the time of observation. Parallax correction is also not applied in form 4 Trav. as in form 11 Topo. This correction should generally be applied in observing to a heavenly body as close to the earth as the sun (vide page 21); but in form 4 Trav. the correction, being small, is not included. Also in form 4 Trav. the N.P.D. computed for Local Apparent noon directly from the decln at apparent noon (given in N.A. p 1), and a correction to the azimuth applied for Dec. changes from the chart in Table 33 Sur instead of the N.P.D. being computed from the Declination interpolated for the exact time of observation as in form 11 Topo. The azimuth in form 4 Trav. is measured from north whereas that in form 11 Topo. is measured from south. Form 4 Trav. has a special line at the end for application of the Convergency from table 11 Sur.

Latitude by Talcott method with the Zenith Telescope.

The zenith telescope is a portable instrument specially adapted for the measurement of small differences of zenith distances. It is the invention of Capt. Talcott, of the United States Corps of Engineers, and has been much used for precise determinations of latitude in the department. For a detailed description reference may be made to the Handbook of Prof[1]. Instruction for the Trigonometrical branch, Part IV; but in its principal characteristics it may be described as having:-

(1) In the eye-piece of the telescope a micrometer capable of measuring small angles to an accuracy of $\frac{1}{20}$ of a second of arc.

(2) A sensitive level by means of which the telescope may be kept at, or nearly at, a constant angle to the vertical during observations both N. and S. of the zenith.

(3) A vertical axis round which the instrument can be revolved to bear N. or S. of the zenith.

Talcott's method of determining latitude by the zenith telescope is as follows:-

A pair of stars is selected, one N. and one S. of zenith, but of nearly equal zenith distances so that they may both be seen in the field of the telescope at one setting of the level, i.e. without altering the altitude of telescope. The R.A. of these two stars should be nearly equal so that their transit may occur within the short period of one another, but leaving sufficient time to read the level and micrometer between the two observations.

With the best value of sidereal time available, the stars are followed by the observer with the micrometer wire until the exact moment of transit, and the level and micrometer read for each one.

Let m	=	micrometer reading of southsstar
m'	=	" " north star
l	=	level correction in arc, of south star
l'	=	" " north "
ζ^o	=	zenith distance of zero of micrometer
$\zeta_,$	=	" " of south star
ζ'	=	" " of north star
r	=	corr. for refraction of south star
r'	=	" " north "

Assuming that the micrometer readings increase as the zenith distances decrease -

Then we have:-

$$\zeta = \zeta^o - m + l + r$$
$$\zeta' = \zeta^o - m' + l + r$$

and
$$\zeta - \zeta' = m' - m + l - l' + r - r' \quad \ldots\ldots\ldots\ldots$$

But if δ and δ' are the declinations of the south and north stars respectively:-

$$\lambda = \delta_, + \zeta_,$$
$$\lambda = \delta' - \zeta'$$
$$\therefore \lambda = \tfrac{1}{2}(\delta + \delta') + \tfrac{1}{2}(\zeta - \zeta') \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{ii}$$

and combining equations i and ii :-

$$\lambda = \tfrac{1}{2}(\delta + \delta') + \tfrac{1}{2}(m' - m) + \tfrac{1}{2}(l - l') + \tfrac{1}{2}(r - r') \quad ..\text{iii}$$

The simplicity of this determination is apparent without any further explanation. Its weak point is that in the choice of pairs of stars it may be necessary to use some stars of which the places are indifferently known.

Both the portable transit, and the transit theodolite may be used as a zenith telescope, if they are furnished with micrometers in the eye-piece.

In the Talcott method it is sometimes necessary to observe the transit of a star over one of the vertical side-wires and reduce the time to what it would have been if the transit had been observed over the centre-wire.

For this purpose it is necessary to find the interval of time which an equatorial star would take to pass from the side wire to the central wire.

To find this interval a circumpolar star fairly low in the sky, of declination δ is observed.

Then Equatorial interval "I" = Observed interval "i" of stars' passing from side wire to central wire x $\cos \delta$.

From this the interval "i'" for a star of different declination δ' to pass from side-wire to centre wire can be determined.

as $i' = I \sec \delta'$

In computing Talcott observations the star places (decl^n & R.A.) may be taken from the N.A., A.E. etc. Sometimes star places which are not given in the Almanacs for all the small stars have to be worked out from a star catalogue such as Newcombs'. A note on the working of star places is here given, as there is some confusion in the notation, which is explained in the following note.

Star Places from Catalogue.

A star as observed is in its apparent place.
Its true place is the above freed from aberration.
Its mean .... nutation.

Secular changes are progressive but slow from year to year and proportional to time during short periods as they take seculae (or centuries) to complete a cycle.

Periodic changes complete their cycle comparatively quickly and are only proportional to time for very short periods.

Precession is due to the slow shift of the first point of Aries (from which Right Ascension is measured, being the intersection of the ecliptic or sun's path and the equator) and is a secular change. Luni-solar precession (due to action of the sun and moon on the protuberance of the equator) causes a motion of the equa-tor along the ecliptic; and planetary motion, a motion of the ecliptic along the equator.

Nutation is an irregularity in the above motion which produces periodic and comparatively small but rapid changes.

Aberration is due to the earth's motion combined with the finite velocity of light, causing an apparent displacement of a star from its true position.

In addition stars have a proper motion which is hardly percepti-ble as a rule and due either to the star actually shifting in space or to the motion of our solar system.

In catalogues the precessions are the values of the changes in R.A. and declination at the period of the catalogue per year or 100 years according to the catalogue used.

Secular variations are the irregularities in the above in 100 years. In some star catalogues, such as Newcomb's, proper motion

(μ) is given both separate and combined with precession (p) under the general heading Centennial Variation (c), where $c = \mu + p$, the unit adhered to being 100 years throughout. Other catalogues use the term annual motion (m) = c/100, μ and p also being given with the year as unit.

To obtain the apparent place of a star, which is what is observed, we use form 3 Lat. The procedure carried out in the form may be explained as follows:-

(1) We take the mean place of the star for the date of the catalogue.

(2) We then apply proper motion precession & secular variation to the mean place to reduce the values to the commencement of the required year.

(3) We then employ Bessel's formulae as modified for any epoch in Turner's tables to reduce star's place to the exact date of the year & time, including the small periodic changes for nutation & aberration necessary to convert the mean to the apparent place.

If m_α be the annual motion of a star in R.A.$=\mu_\alpha + p_\alpha = \frac{d\alpha}{dt}$

s_α be the secular variation " " " $= 100 \frac{d^2\alpha}{dt}$

α_0 be the mean R.A. for time t, or date of catalogue

α .... t_y , y years from it

$$\text{R.A.} = \alpha = \alpha_0 + y\left[m_\alpha + \frac{s_\alpha}{100}\,\frac{y}{2}\right] \text{......} \quad (1)$$

Similarly, using corresponding symbols and subscript letters d for the annual motion and secular variation in N.P.D:-

$$\text{N.P.D.} = d = d_0 + y\left[m_d + \frac{s_d}{100}\,\frac{y}{2}\right] \text{......} (2)$$

If the catalogue gives Declination instead of N.P.D. , the signs of m_d & s_d must be reversed.

The results (1),(2) give the mean places of a star at the commencement of the particular year for which its apparent place is required. (vide the first 15 lines of form 3 Lat).

It now remains to apply Bessel's formulae, as modified for any epoch, in Turnerls Tables, inorder to obtain the apparent place.

The formulae for these corrections, given on p V of the introduction to Turner's Tables, (including $t\mu$, $t\mu'$ to bring the star places up to the actual date and time t, from the commencement of the year), are:-

$$\begin{aligned} &\text{Correction to R.A. (1)} = t\mu + Aa + Bb + (nC)c + Dd + f \\ &\text{.......... N.P.D. (2)} = t\mu' + Aa' + Bb' + (nC)c' + Dd' + i.sa' \end{aligned} \quad \text{.... (3)}$$

These formulae are merely modifications of those used in the Nautical Almanac. The day-numbers A, B, C, D however are not in Bessel's, but in Baily's notation, which was definitely abandoned by the Nautical and other almanacs, which reverted to Bessel's notation in 1916.

Baily started his confusing notation in the British Association star catalogue for no better reason than that he considered it formed a good 'memoria technica' if the Bessel factors were altered so that A, B represented the quantities whereby ABerration was determined, C those whereby preCession was determined and D those whereby Deviation (nutation) was determined (vide Doolittle's Astronomy p 616 footnote).

Thus in order to bring Turner's formulae to accord with the present Bessel's notation of the Nautical Almanac, A has to be interchanged with C and B with D. The formulae of form 3 Lat., which are adopted for Turner's Tables thus become:-

$$\begin{aligned} &\text{Correction to R.A. (1)} = t\mu + Ca + Db + (nA)c + Bd + f \\ &\text{.......... N.P.D. (2)} = t\mu' + Ca' + Db' + (nA)c' + Bd' + i.sa' \end{aligned} \quad (4)$$

In the formulae (4) A,B,C,D represent the Nautical Almanac "Bessel's day numbers and f & i other factors (computed from the formulae on pages 631-34 N.A. 1925). which depend on the moon and sun's longitude, that of their perigee, moon's ascending node, obliquity of the ecliptic, etc. They are published in the N A. for each day of the year e.g:- N.A. 1930 p 211-26.

The formulae (4) may be written out in full as:-

$$\text{Correction to R.A. (1)} = t\mu + C\overbrace{\left(\tfrac{1}{15}\sec\delta\right)}^{a}\overbrace{\cos\alpha}^{a} + D\overbrace{\left(\tfrac{1}{15}\sec\delta\right)}^{b}\overbrace{\sin\alpha}^{b}$$

$$+ A\overbrace{\left(\tfrac{1}{15}\tan\delta\right)}^{c}\overbrace{\sin\alpha}^{c} + B\overbrace{\left(\tfrac{1}{15}\tan\delta\right)}^{d}\overbrace{\cos\alpha}^{d} + \underset{c'}{f}$$

$$\ldots\ldots\ldots\ldots \text{N.P.D. (2)} = t\mu' + C\overbrace{\sin\delta}^{a'}\overbrace{\sin\alpha}^{a'} + D\overbrace{\sin\delta}^{b'}\overbrace{\cos\alpha}^{b'} + (nA)\underset{d}{\cos\alpha}$$

$$+ B\sin\alpha + i\,\underset{sa}{\cos\delta} \quad \ldots\ldots (5)$$

Turner utilises the factors

$$f = \left[3\overset{s}{.}07234 + 0\overset{s}{.}00186\,\frac{t}{100}\right]A$$

$$nA = g\cos G = \left[20''.0468 - 0''.0085\,\frac{t}{100}\right]A$$

$$i = C\tan\omega, \text{ where } t \text{ is reckoned from 1900,}$$

(vide p 633 N.A. 1925 and Turner's Tables Introduction p IV & V) and thus his Tables are conveniently adapted to suit any epoch.

In his N.P.D. Tables he tabulates

$$\left.\begin{aligned} a &= b = \tfrac{1}{15}\sec\delta \\ c &= d = \tfrac{1}{15}\tan\delta \\ a' &= b' = \sin\delta \\ sa' &= \cos\delta \end{aligned}\right\} \ldots\ldots (6)$$

In his R.A. Tables he tabulates

$$\left.\begin{aligned} &\sin\alpha \text{ as } b, c, a', d' \\ &\cos\alpha \text{ and } a, d, b', c' \end{aligned}\right\} \ldots\ldots (7)$$

By comparing (6) & (7) with (5) the remainder of the computation on form 3 Lat. will be easily intelligible from the form itself together with the explanations in the footnotes.

The fictitious year

We have hitherto spoken of the year without definitely stating which of the various periods called a year was to be understood. Neither the common year (with every fourth year a leap year) nor the Julian year of 365¼ days is well adopted for astronomical calculations so Bessel introduced the fictitious year to obviate the difficulties which would arise from using the common or Julian year in these computations.

The fictitious year commences when longitude of mean sun is $\underline{\underline{280}}^{\circ}$ (or R.A. = 18h 40m)

By this device simplicity is obtained in quantities which are functions of T. This is the date for which mean places are reduced in star catalogues.

The annual precession given in star catalogues is for a mean year of 365 days 5.8 hours.

Catalogues give values of T or its logarithm reckoned from the commencement of the fictitious year and reduced to decimal parts of the mean tropical year.

In tables containing values of A,B,C,D, the argument is the sidereal date at the fictitious meridian.

To obtain this date it is to be observed that the tables are immediately available on the fictitious meridian for the sidereal time 18h. 40m., without any reduction of the date.

For any other meridian at the sideral time 18h. 40m. the argument of the table will be the reduced date, but at any other sidereal time g the argument must be this reduced date increased by $\frac{g - 18h\ 40m.}{24h}$ which must be always taken < 1 and positive or by

$$g' = \frac{g + 5h\ 20m}{24h}, \text{ omitting one whole day if } g + 5h\ 20m \geqq 24h.$$

Latitude and time by the Prismatic Astrolabe.

The principle of this observation is as follows:- If any 3 or more stars are observed to reach the same altitude at times which are noted, then it is possible to calculate (1) the altitude at which the stars were observed (2) the chronometer error (3) the latitude. It is usual to observe groups of 4 stars, one in each quadrant of the heavens, N.E., S.E., S.W., N.W.

The instrument consists of a horizontal telescope with an equilateral glass prism fixed in front of the object end, with the side next the object glass vertical. Under and in front of the prism is placed an artificial horizon, viz:- a shallow dish of mercury. The instrument is constructed to observe stars at a constant elevation of about 60°, and the time they reach this elevation is noted by means of an observation of the coincidence of 2 images of the star, one coming direct from the star through the upper inclined face of the prism and refracted to the eye of the observer, the other coming from the star's reflection in the artificial horizon through the lower inclined face of the prism and also refracted to the eye of the observer. An electric chronograph, stop watch, or other accurate means of noting the times is essential. A list of the more important stars for observation are given in a book on the Prismatic Astrolabe by Messrs. J. Ball & Knox Shaw and a programme (on forms 1 & 2 Ast.) can be made up from this or from the special diagram by Mr. J. Ball. The computation of the azimuths and times at which the stars attain the elevation of 60° approx., the constant angle for the instrument, is a fairly simple one, vide form 3 Ast.

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[illegible] consists of a horizontal telescope with an

" 54 line 16 & 19 from top, for refracted read reflected.

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We have alt $h = 60^\circ$ (approx), Z.D. $\zeta = 30^\circ$ (approx), N.P.D. $= \Delta$

Colat. $= \gamma$ (rough value assumed)

$$\tan \frac{t_0}{2} = \sqrt{\frac{\sin(s-\Delta)\sin(s-\gamma)}{\sin s \sin(s-\zeta)}}$$

$$\tan \frac{A}{2} = \sin(s-\zeta)\ \mathrm{cosec}\ (s-\Delta)\tan\frac{t_0}{2}$$

also t_0 in arc x 15 = t_0 in time.

The azimuths of the 4 stars are plotted in the 4 quadrants on a diagram paper from a point O as shown and perprs. drawn to these directions through pts $t_1 t_2 t_3 t_4$ representing the time errors (derived from differences between computed and observed times) plotted to scale from the line of assumed watch error O N. e.g.:- If the

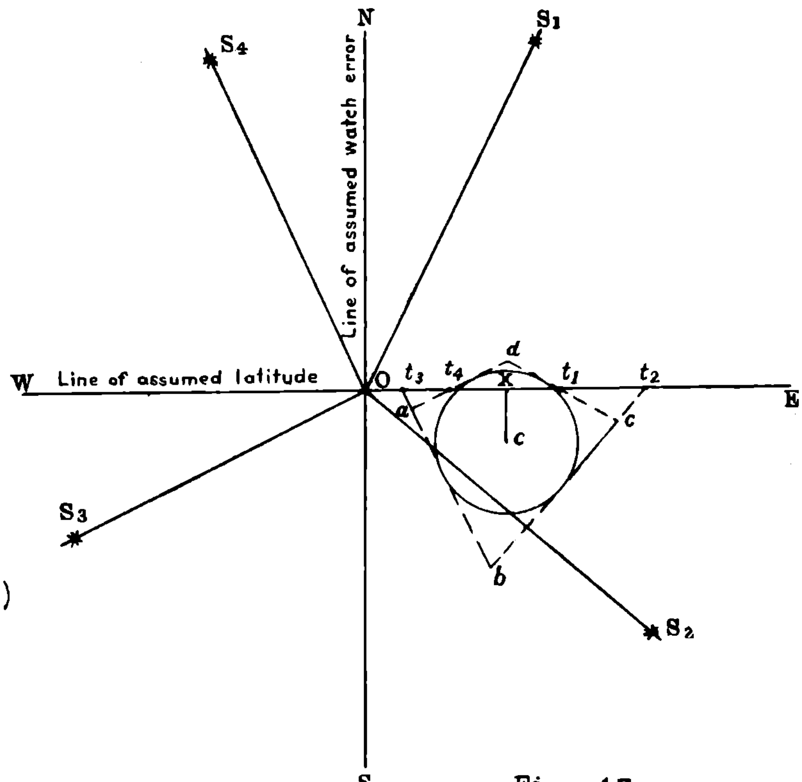

Fig. 17

watch error is known to be about 1 m and some odd seconds, we can assume the error as 1 minute and only plot the odd seconds, so as not to carry our diagram to an inconvenient distance from O, the centre of the paper.

The lines so drawn either meet in a point or form a quadrilateral in which a circle can be inscribed. The x coordinate of the

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point, or of the centre of the circle, divided by 15, is the correction to the estimated time;and the (y coordinate) x (cos λ), the correction to the rough value of latitude assumed at the commencement. With a little practice time should be determinable within 1/10th of a second and latitude within about 1" by about a couple of hours observation.

Form 4 Ast. is intended for use, when observations go on from day to day and adapts computations on 3 Ast. for one day to successive days, so as to avoid recomputation. R.A. is also computed here. Form 5 Ast. combines forms 3 & 4 Ast. and gives the L.S.T. of observations.

The above completes a brief description of the graphic method of using the Prismatic Astrolabe for latitude and time observations. Forms 6 to 10 Ast. are intended for combining observations, obtaining probable errors, errors of clock rates etc., and are applicable more especially when rigorous and not graphic methods are employed.

An explanation of these is outside the scope of these notes.

Determination of Longitude.

The methods of determination of longitude will here only be briefly described, as these notes only aim at giving the details of those field astronomical observations which the Topographical Surveyor is likely to have to carry out ordinarily. Differences of longitude correspond to differences of time. We have already shown how to obtain the local time of a place by astronomical observation, and we merely have to know the local time of the place of reference at the same instant for which we know our own local time, to be able to compare the longitude of the place of observation with the longitude of the place of reference. Each hour of time = 15° of longitude.

The place or meridian of reference usually employed is Greenwich, but it may be any other place or meridian where we are enabled to obtain the longitude and true time, and the ascertaining of the relative values of the two local times at any two stations for any one particular instant constitutes the whole difficulty of this particular section of practical astronomy.

The principal methods of determining longitude are:-

(1) Electric telegraph (2) Wireless signals (3) Transport of chronometers (4) Triangulation (5) Traverse (6) Latitude and azimuth.	Relative methods
(7) Occultation of stars by moon (8) Eclipse of sun by moon (9) Moon culminating stars (10) Moon photographs (11) Moon altitudes (12) Lunar distances (13) Eclipse, occultation, or transit of Jupiter's satelites.	Absolute methods.

In the first 2 methods, differences of longitude are obtained by comparing the correct local time at one station with that at the other, by transmission of signals between the stations, certain corrections being necessary for personal equation between the observers, clock rates, rates of wireless transmission, etc. For details of method (1), vide old Hand book of Prof[l]. Instructionsfor the Trig[l]. branch 1902, p 75; also Topo Chapter VII 1924, para 21. In the third method a number of chronometers are transported between places, the accurate determination of difference of longitude between which is required, and the mean results given by these chronometers taken for comparison with the local time.

When moving from place to place with chronometers they develope travelling rates, and these require to be carefully determined and applied. If the camp halts, the chronometer should be sent out for a normal day's march during the days of halt, or, if the halt is extended, the rate at rest should be determined.

Methods (4) and (5) are normal methods, (vide Topo Hand book, Chapters III & IV), and require no explanation here.

Method (6) is described in Topo Chapter VII 1924, paras 25 (a) and (b).

The remaining methods (7) to (13) for determining longitude depend on the observation of certain phenomena (either of effect or of the relative positions of the celestial bodies), the Greenwich time of which can be calculated from the data given in the Nautical Almanac and compared with the time at the place when the phenomena were observed, so as to obtain the longitude.